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ON THE DESIGN OF LINEAR PROCESS ADAPTIVE CONTROL SYSTEMS

by

Harry Nathan Yagoda

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POLYTECHNIC INSTITUTE OF BROOKLYN
MICROWAVE RESEARCH INSTITUTE
ELECTRICAL ENGINEERING DEPARTMENT

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AN ABSTRACT

ON THE DESIGN OF LINEAR PROCESS
ADAPTIVE CONTROL SYSTEMS

by

Harry Nathan Yagoda

This dissertation treats the problem of controlling a stable, slowly time varying, linear process. A solution is proposed that is simple, practical and applicable to any finite order process. Use is made of a continuously adjusted tandem compensator; a comparison of the weighted histories of the input and output signals is used to control this adjustment. System design is based upon the use of a variable transfer function characterization for both the compensator and the process. In effect, a compensator is designed that attempts to cancel the variable poles of the process with variable zeros, the variable zeros of the process with variable poles and the variable process gain with its reciprocal.

The design technique developed is applied to several problems. The processes involved range from a variable gain amplifier through a plant that contains a variable gain and two pair of variable complex poles. The processes are grouped in accordance with the number of variable parameters in the process characterizing function.

Several adaptive processes are simulated on a computer and the resulting operation of the adaptive circuitry is presented for comparison with the theoretically predicted operation. Among the processes investigated are:

1. a variable gain amplifier
2. a process with two variable real poles
3. a process with a variable gain and two pair of variable complex poles.

The adaptive circuitry is modified for the process with two variable real poles and the resulting operation of that adaptive process is presented. The agreement between the theoretically predicted results and the operation of the simulated systems is good.

The amenability to analysis of the technique presented, allows for the investigation of the effect of noise on the adaptive process. In addition, stability is investigated. Neither of these appears to offer a major problem in the operation of the proposed control systems.

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CHAPTER 1

Introduction

Considerable attention has been devoted during the past five years to the design of adaptive control systems. This work is being pursued in two directions. The first of these considers the problem of maintaining a specified performance in the presence of input parameter variations. Systems designed using this viewpoint are termed signal adaptive control systems. The second problem considers maintaining a specified performance in the presence of process parameter variations. Systems designed using this viewpoint are termed process adaptive control systems. The work in this dissertation deals with the second of these problems under the restriction that the process is a stable, slowly time varying, linear process. The technique developed is intended for use in those problems which, due to excessively large process variation, cannot be handled adequately using feedback theory.

A process in the class considered is described by the linear differential equation that relates its output $c(t)$ to its input $r(t)$. The coefficients of this equation are time dependent; the constraint of "slowly time varying", however, is taken to mean that the percentage variation in any coefficient during a time interval of length T is negligibly small (T is assumed much greater than the largest process response time). Thus over an interval of length T the process is handled as if it were fixed.

Most of the process adaptive control system design techniques that exist apply only to processes in the class considered. They are divided according to whether or not they contain a separable control computer and subdivided according to the intended function of the adaptive control circuitry (i.e., to maintain a fixed transfer characteristic between input and output or to minimize some given error criterion).[1] There are many examples of each of these approaches.[2,3,4] Most suffer from one of two problems: either they are highly impractical or they are highly specific and not applicable in general.

The approach used in this dissertation is to construct adaptive circuitry that operates to maintain a fixed transfer characteristic; the intent is to develop a method for adaptive process design that is simple, general and gives a practical solution. The resulting technique satisfies these requirements and yields a control system, for any order

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[1]E. Mishkin and L. Braun, "Adaptive Control Systems", McGraw Hill, N.Y., (1961).

[2]J. A. Aseltine, A. R. Manicini, and C. W. Sarture, "A Survey of Adaptive Control Systems", IRE Trans. on A.C., pp. 102-108, Dec. 1958.

[3]P. C. Gregory (ed.), Proc. of the Self-Adaptive Flight Control Systems Symposium, WADC Tech Rept. 59-49, ASTIA Document AD209389, Wright-Patterson Air Force Base, Ohio, March 1959.

[4]P. R. Stromer, Adaptive or Self-Optimizing Control Systems - A Bibliography, IRE Trans. on Automatic Control, vol. AC-6, pp. 65-68, May 1959.

process, that tends to maintain a fixed transfer characteristic via parameter tracking. Figure 1-1 shows a block diagram of the general system to be considered. The variable linear tandem compensator is used to compensate the forward transmission path for changes in the process. Adjustment of the compensator is continuous and based upon a comparison of the weighted histories of the input and output signals. The adjustment signals tend to drive any differences toward zero.

The compensator is designed expressly for the process. It is equipped with one variable zero for each of the variable poles of the process transfer characteristic, one variable pole for each of the variable zeros of the process transfer characteristic, and a variable gain if required. Thus each of the variable parameters of the compensator is matched to one of the variable parameters of the process. The function of the adjusting circuitry is then to cause parameter tracking. Although other process adaptive techniques exist that employ some sort of tandem compensation and parameter tracking, most of these are limited, either

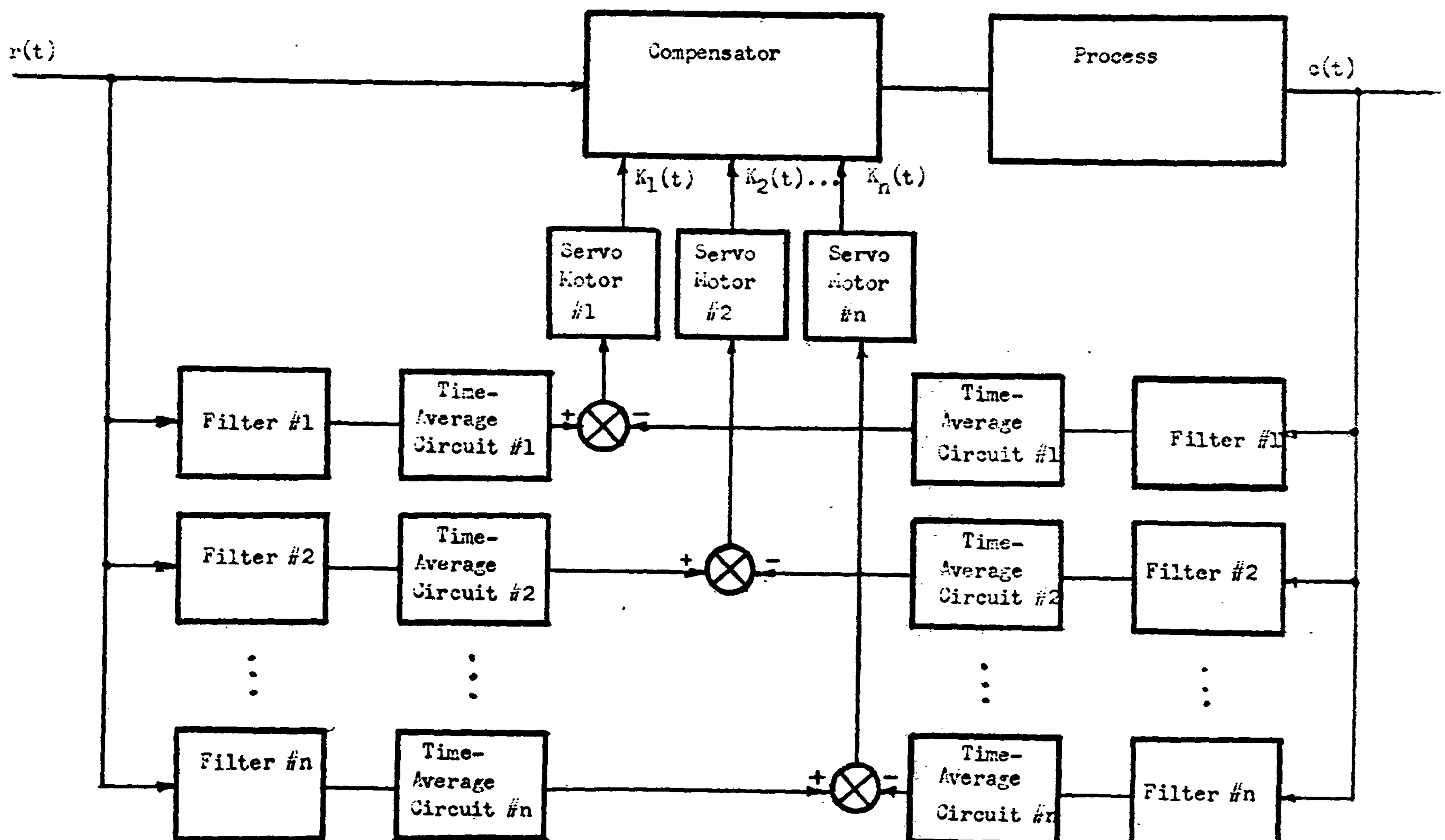

Figure 1-1 General Block Diagram of Linear Process Adaptive Control System

theoretically or practically, to one or two parameters.[5,6,7,8,9] The design technique presented here is applied to several important multi-variable parameter processes and the results of some computer simulations are presented.

The problem of designing a process adaptive control system is broken into three parts. The first involves choosing an appropriate characterization for the process. A transfer function with variable coefficients is selected. Next, a tandem compensator is designed. This is also done using a variable coefficient transfer function characterization. Then a criterion for adjusting the compensator is selected. The first portion of the paper is devoted to these efforts. The technique that is developed is then applied to several particular problems, including one that appears impossible using feedback theory.

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[5] R. M. Corbin and E. Mishkin, On the Measurement Problem in Adaptive Systems Utilizing Analog Computer Techniques, Research Report 12-699-58, PIB 627, Poly-Inst. of Brooklyn, MRI, 1958.

[6] R. Staffin, "Executive-Controlled Adaptive Systems", doctoral dissertation, Polytechnic Institute of Brooklyn, 1958.

[7] M. Margolis and C. T. Leondes, "A Parameter Tracking Servo for Adaptive Control Systems", IRE Trans. on Automatic Control, Vol. AC-4, No. 2, November 1959.

[8] C. N. Weygandt and N. N. Puri, "Transfer Function Tracking and Adaptive Control Systems", IRE Trans. on Automatic Control, Vol. AC-6, No. 2, May 1961.

[9] H. P. Whitaker, J. Yamron and A. Kezer, "Design of Model-Reference Adaptive Control Systems for Aircraft", Report R-164, MIT Instrumentation Laboratory, September 1958.

CHAPTER 2

The Process

One of the most important attributes of an adaptive process control system design technique is the manner in which the process is characterized. This directly affects the form of the compensation, as well as the mathematical difficulty involved in analyzing the model. It is for these reasons that a process characterization using a transfer function with variable coefficients is chosen. The error involved in selecting such a characterization is discussed.

2.1 Differential Equations

The behavior of a process in the class being considered is completely characterized by a differential equation of the form:

$$\sum_{i=0}^{n} a_i(t) \frac{d^i}{dt^i} m(t) = \sum_{j=0}^{m} b_j(t) \frac{d^j}{dt^j} c(t) \qquad (2\text{-}1)$$

Using this characterization it is possible to specify a tandem compensator with which to achieve an error-free control system. This compensator is characterized by the differential equation

$$\sum_{j=0}^{m} b_j(t) \frac{d^j}{dt^j} r(t) = \sum_{i=0}^{n} a_i(t) \frac{d^i}{dt^i} m(t) \qquad (2\text{-}2)$$

Unfortunately the functions $a_i(t)$ and $b_j(t)$ are not all known beforehand and thus this compensator cannot be constructed. No other compensator gives an error-free control system. As a result, an approximate characterization is useful providing it leads to a control system that is satisfactory and simply constructed.

For a large class of control signals m(t) applied to a process, the output c(t) is very closely approximated over a time interval of length T preceding the time t_o by the solution of[1]

$$\sum_{i=0}^{n} a_i(t_o) \frac{d^i}{dt^i} m(t) = \sum_{j=0}^{m} b_j(t_o) \frac{d^j}{dt^j} r(t) \tag{2-3}$$

Thus it is possible to approximate the output of a variable process during the T seconds preceding time t_o by the output of a fixed process during that interval, providing the parameters of the fixed process are properly chosen; the fixed process parameters are taken equal to the variable process parameters at the time instant t_o.

A variable process is thus characterized approximately by a family of fixed processes, one for each instant of time. Since each of these fixed processes is characterized by a transfer function $G_j(s)$, the variable process is

[1]Based on the restriction that the process is slowly varying.

characterized by a family of transfer functions and is represented as a single transfer function with variable coefficients. This is defined to be the "process characterizing function" and is denoted as $G(s,t)$.

Use is made of the approach presented above in selecting a tandem network with which to compensate the process. The desired control system transfer function $G(s)$ is divided by $G(s,t)$ and the resulting function, denoted by $G_c(s,t)$, is defined as the "compensator characterizing function". This function then characterizes the network that is used for compensation. At any instant the two characterizing functions represent a pair of transfer functions; the product of these transfer functions is the desired control system transfer function. Process compensation is thus accomplished by compensating each of the transfer functions in the family characterizing the process by the corresponding transfer function in the family characterizing the compensator.

For physical systems there is a lack of complete information concerning future variations in the process and thus the process characterizing function is unknown beforehand. A compensator is therefore used for which the compensator characterizing function is experimentally generated on the basis of measurements made on the compensated process. As a result there are two errors present in this formulation for the control system: one is due to the manner in which the process and the compensator are characterized and the other is due to the need for experimental determination of the compensator characterizing function. To discussing these errors it is necessary to consider the impulse response of the variable process. This is done next.

2.2 Impulse Response

The behavior of a process in the class considered is completely characterized by an impulse characterizing function $g(t_1,t_2)$ where t_1 is the time the impulse is applied to the process and t_2 is the time elapsed since t_1. The response of the process to an impulse applied at time t_a is therefore $g(t_a, t-t_a)$. The behavior of a fixed process is also characterized by an impulse characterizing function $g_f(t_1,t_2)$. The response of this process to an impulse applied at time t_a is $g_f(t_a, t-t_a)$; this function does not depend on t_a and is written as $g_f(t-t_a)$.

For the processes being considered $g(t_o, t-t_o)$ is approximately equal to $g_{fo}(t-t_o)$, where $g_{fo}(t-t_o) = \mathcal{L}^{-1}[G(s,t_o)]$; in fact $g(t_a, t-t_a)$ is approximately equal to $g_{fo}(t-t_a)$ for any t_a such that $|t_a-t_o| < T$. Thus the response of the process to any impulse that is applied during a period of length T prior to the time t_o is approximately $g_{fo}(t-t_a)$. It is this approximation that is used in adjusting the compensator; in effect, this corresponds to replacing the variable impulse response function by a fixed impulse response function during the interval between t_o-T and t_o.

The impulse response functions discussed above are represented as surfaces in three dimensional space, as shown

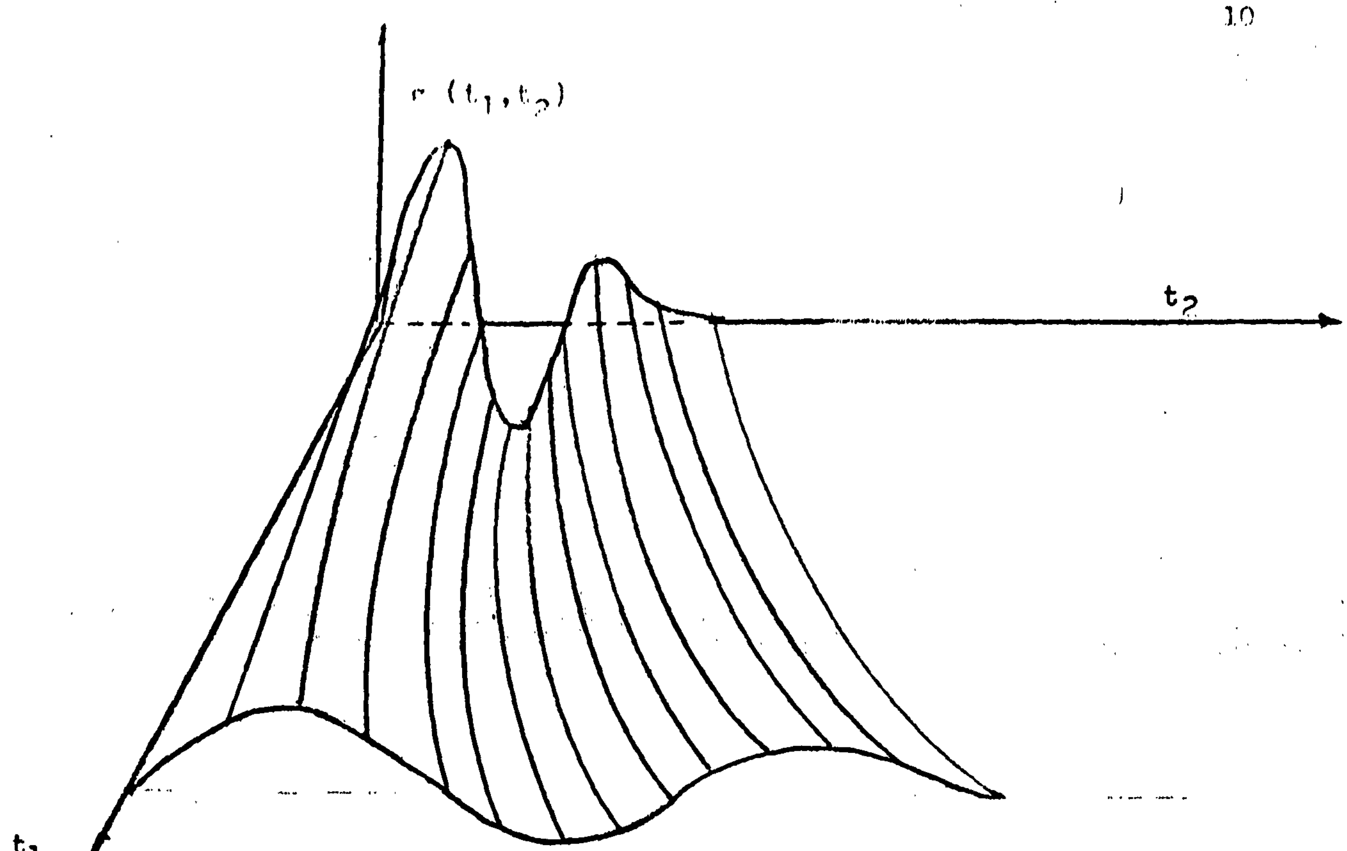

Figure 2-1a Variable Process Impulse Response Function

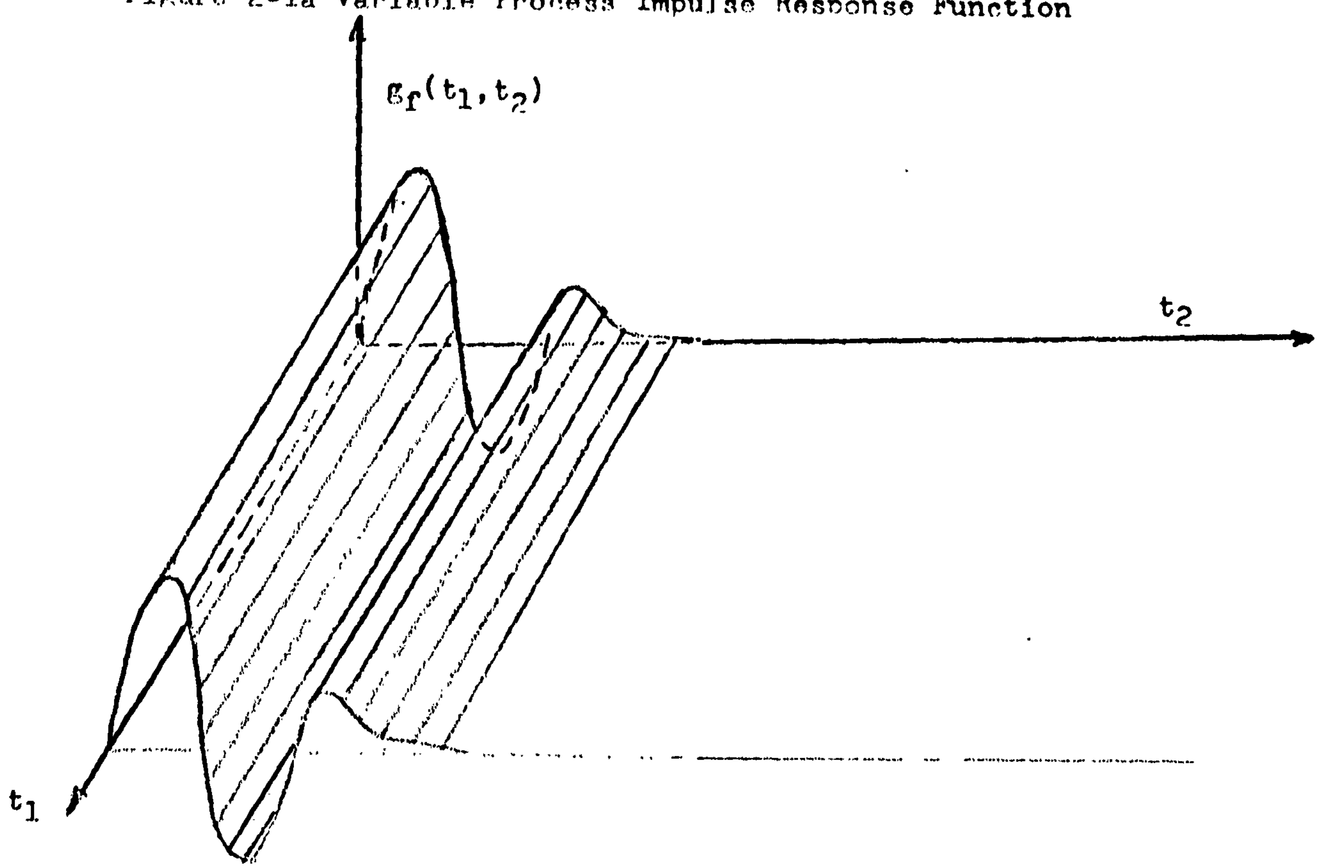

Figure 2-1b Fixed Process Impulse Response Function

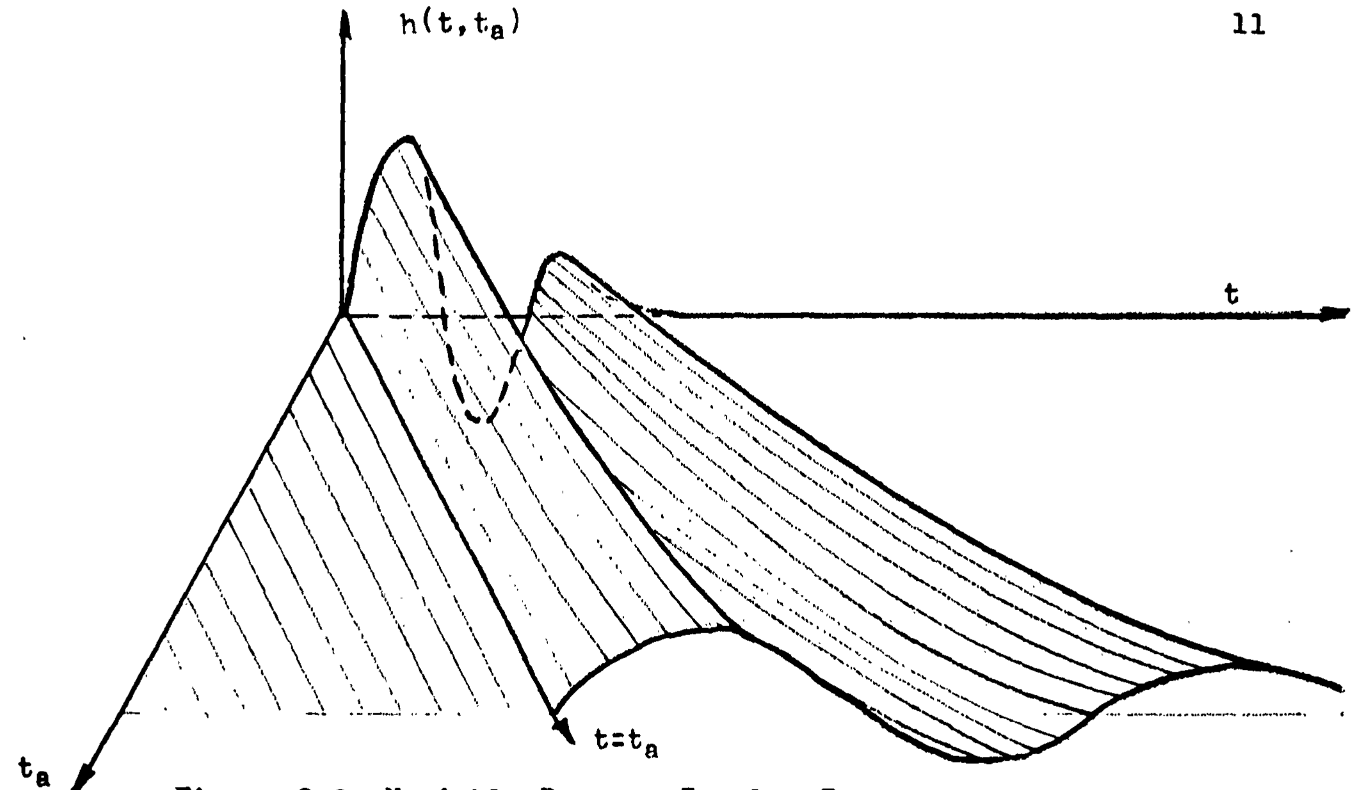

Figure 2-2a Variable Process Impulse Response

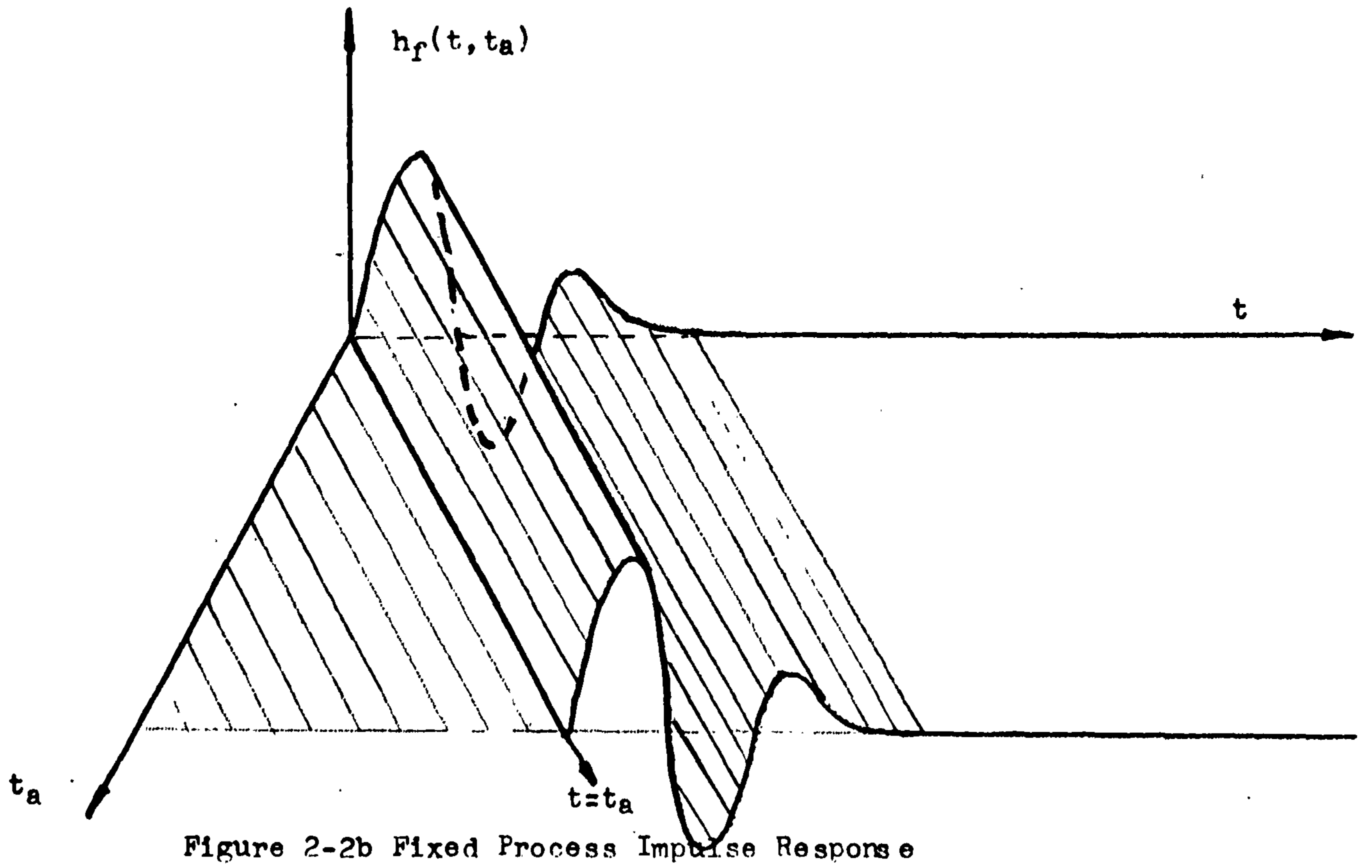

Figure 2-2b Fixed Process Impulse Response

in figures 2-1a, 2-1b, 2-2a and 2-2b. The response of the fixed process to an impulse of amplitude m applied at time t_o is $mg_{fo}(t-t_o)$; the response of this process to an input of m(t) is then given by:

$$C_{fo}(t) = \int_{-\infty}^{\infty} m(t_a)\, g_{fo}(t-t_a)\, dt_a$$

$$= \int_{-\infty}^{t_o-T} m(t_a)\, g_{fo}(t-t_a)\, dt_a$$

$$+ \int_{t_o-T}^{t} m(t_a)\, g_{fo}(t-t_a)\, dt_a \qquad (2\text{-}4)$$

since $g_{fo}(t-t_a)$ equal zero for t_a greater than t. The response of the variable process to an impulse of amplitude m applied at time t_a is $mg(t_a, t-t_a)$; the response of this process to the input m(t) is given by:

$$C(t) = \int_{-\infty}^{\infty} m(t_a)\, g(t_a,\ t-t_a)\, dt_a$$

$$= \int_{-\infty}^{t_o-T} m(t_a)\, g(t_a,\ t-t_a)\, dt_a$$

$$+ \int_{t_o-T}^{t} m(t_a)\, g(t_a,\ t-t_a)\, dt_a \qquad (2\text{-}5)$$

since $g(t_a,\ t-t_a)$ equal zero for t_a greater than t.

The output $C(t)$ of the variable process and the output $C_{fo}(t)$ of the fixed process are now compared for a period of time prior to t_o. The difference between the two is defined as the characterization error and equations 2-4 and 2-5 are used to evaluate this error.

2.3 Discussion of the Characterization Error

Consideration is now restricted to the time interval between $t_o-t + 5t_r$ and t_o; t_r is the largest of the response time constants of the process characterizing function. The characterization error is then defined to be the difference between the output of the process and the output predicted by using a fixed process; this fixed process corresponds to the process characterizing function evaluated at t equal t_o. This error is denoted by $\varepsilon(t)$ and is given by:

$$\varepsilon(t) = C(t)$$

$$- C_{fo}(t) = \int_{-\infty}^{t_o - T} m(t_a)\, g(t_a,\, t-t_a)\, dt_a$$

$$- \int_{-\infty}^{t_o - T} m(t_a)\, g_{fo}(t-t_a)\, dt_a$$

$$+ \int_{t_o - T}^{t} m(t_a)\, [g(t_a,\, t-t_a) - g_{fo}(t-t_a)]\, dt_a$$

(2-6)

$m(t_a)$ is then divided into two parts; the first is that portion of $m(t_a)$ that occurs before $t_a = t_o - T$ and it is denoted as $m_1(t_a)$. The second part is that portion of $m(t)$ that occurs after $t_a = t_o - T$; it is denoted as $m_2(t_a)$. Equation 2-6 is then rewritten as:

$$\varepsilon(t) = \int_{-\infty}^{t_o - T} m_1(t_a)\, g(t_a,\, t-t_a)\, dt_a$$

$$- \int_{-\infty}^{t_o - T} m_1(t_a)\, g_{fo}(t-t_a)\, dt_a$$

$$+ \int_{t_o - T}^{t} m_2(t_a)\, [g(t_a,\, t-t_a) - g_{fo}(t-t_a)]\, dt_a$$

(2-6a)

The first integral in equation 2-6a represents the output of the variable process due to the energy storage caused by $m_1(t_a)$. Since t occurs more than five time constants after $m_1(t_a)$ becomes identically equal to zero, this integral is negligibly small when applied to physical processes that contains noise. The second integral represents the output of the fixed characterizing process due to the energy storage caused by $m_1(t_a)$. This integral is also negligibly small. The third integral represents the error in the outputs caused by $m_2(t_a)$; it is a result of the difference between the impulse responses of the fixed and variable processes. This contribution to the error is likewise negligibly small since it relates directly to the definition of a slowly varying process.[2] The more slowly varying the process, the smaller the error.

2.4 Conclusions

The process is characterized by a family or set of transfer functions. This set is denoted in parametric form, with time as the parametric variable [i.e., G(s, t)]. For any given time the characterization gives a transfer function that is usable in calculating the process output during a time interval preceding that time. In addition, a similar characterization is indicated for a tandem compensator. Using this characterization a tandem compensator is investigated.

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[2] This in effect is the defining criterion for a slowly varying process.

CHAPTER 3

The Compensator

Based on the characterization chosen for the process a characterization for a tandem compensator is indicated. This characterization is further developed and a general method for constructing the compensator is presented. The approach used is to build all compensators from five basic compensating elements. It is assumed that the region over which a given parameter varies is known; if this is not so, an allowable region over which the parameter is to be compensated is chosen.

3.1 General

For a desired control system transfer function of $G(s)$ and a process characterizing function of $G(s,t)$, the compensator characterizing function $G_c(s,t)$ is given by:

$$G_c(s,t) = \frac{G(s)}{G(s,t)} \tag{3-1}$$

Since both $G(s)$ and $G(s,t)$ are functions of s, equation 3-1 is rewritten after factoring as:

$$G_c(s,t) = K_c \frac{(s+b_1)\cdots(s+b_n)(s^2+c_1s+d_1)\cdots(s^2+c_ms+b_m)}{(s+e_1)\cdots(s+e_p)(s^2+f_1s+g_1)\cdots(s^2+f_qs+g_q)} e^{as} \tag{3-1a}$$

Any of the coefficients in equation 3-1a may depend on time.[1] The problem of constructing a compensator is thus reduced to a network synthesis problem. Prior to the investigation of this problem, however, $G_c(s,t)$ is restricted.

Variations in the coefficient "a" are not amenable to correction by the system presented; this is due to the dependence on amplitude rather then phase information of the compensator control circuitry. Variable terms of the form e^{as} are omitted from further consideration. As a result, it is at most necessary to synthesize a compensator characterizing function that contains a variable gain, variable poles and variable zeros. This is done by cascading a number of individual compensator elements; each of these elements compensates the process for just one variable term. A general compensator is shown in figure 3-1.

3.2 Variable Gain

There are many ways of constructing a compensator element for the variable-gain term. The method presented here consists of cascading a potentiometer with a constant gain amplifier; the gain of the amplifier is made equal to the maximum expected value of K_c. Compensator gains below this value then correspond to settings of the potentiometer

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[1] It is assumed that the poles of $G_c(s,t)$ remain in the left half s-plane.

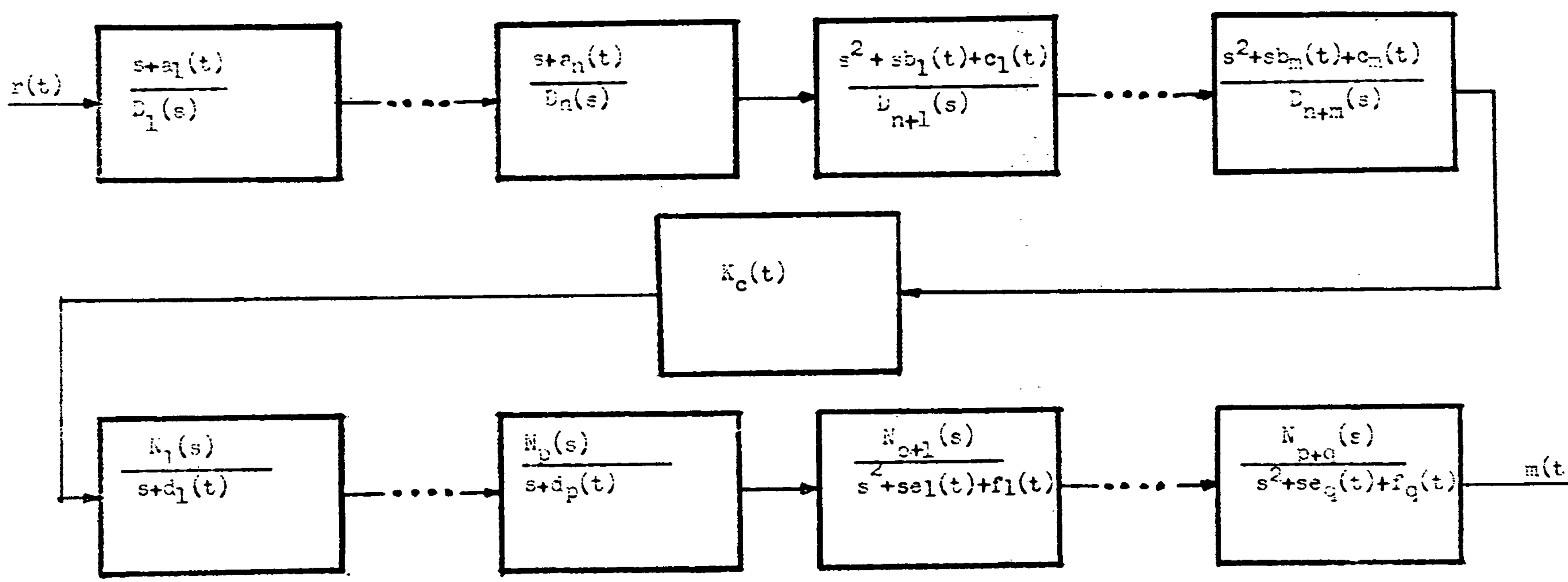


Figure 3-1 General Compensator

between zero and one. Adjustment of the gain compensating element is then made by positioning the potentiometer; this positioning is done by a small motor.

3.3 A Variable Real Zero

To compensate a process that contains a variable real pole in its characterizing function, it is necessary to synthesize a compensator characterizing function that contains a variable real zero. This synthesis necessitates the use of a compensator element that contains a variable real zero. Such an element is constructed as follows: two fixed networks, each in series with a variable loss, are placed in parallel as is shown in figure 3-2a. The fixed networks are then chosen to be identical except for one zero; in each of the networks this zero is chosen to correspond to one of the extreme values in the position of the pole being compensated. As a result the characterizing function of the compensator element is identical to either of the fixed networks, except for the variable zero. The position of this variable zero is somewhere between the two zeros of the fixed network.

The characterizing function for a typical compensator element is given in equation 3-2 and a root locus plot for it is shown in figure 3-3.

$$G_1(s,t) = \frac{s + [a+(b-a)K_2(t)]}{s + d} \tag{3-2}$$

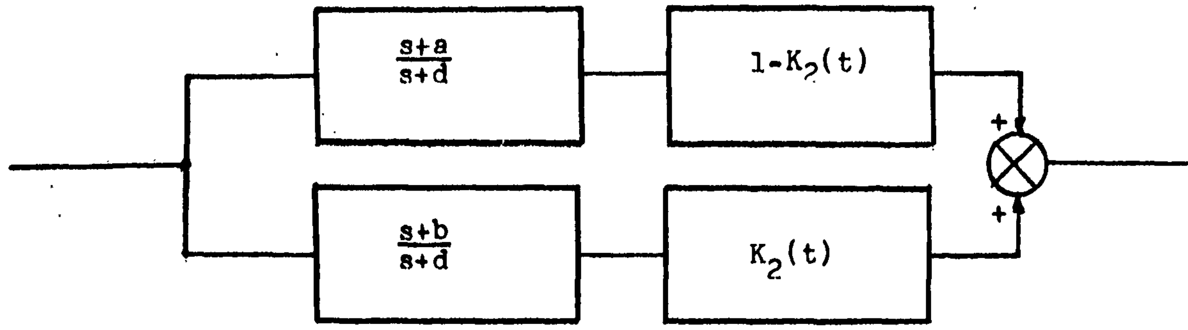


Figure 3-2a Compensator Element Containing a Variable Real Zero

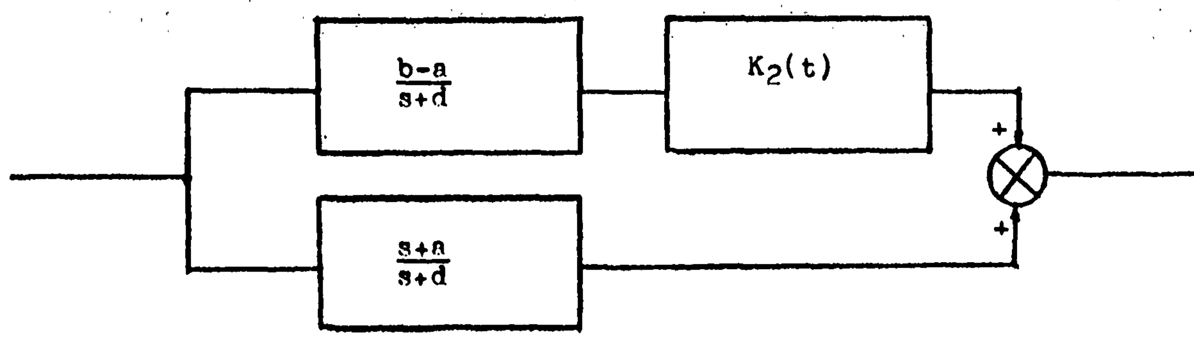
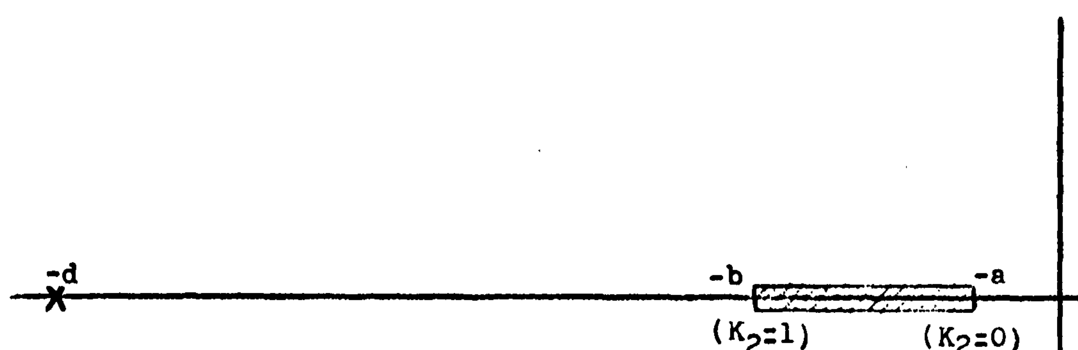


Figure 3-2b Alternate Compensator for Variable Real Zero



$$G\ (s,t) = \frac{s + a + (b-a)K_2(t)}{s+d}$$

Figure 3-3 Root Locus Plot for a Variable Zero

From equation 3-2 it is noted that the position of the variable zero is dependent upon a loss that varies between zero and one. An alternate circuit configuration that gives this characterizing function and requires only one motor that positions one potentiometer is shown in figure 3-2b.

3.4 A Pair of Variable Complex Zeros

In constructing a compensator for a process that contains a pair of variable complex poles in its characterizing function it is necessary to use a compensator element that contains a pair of variable complex zeros in its characterizing function. In the construction of such an element, two approaches are considered: the first is based upon the separate adjustment of the real component α and the imaginary component β of the zeros; the second is based upon the separate adjustment of the natural frequency ω_n and the damping ξ of the zeros. In both cases two independent parameter adjustments are required since two degrees of freedom are involved. In addition, corresponding limits are required for the maximum expected values of these parameters.

The construction of either of the above compensator elements requires the use of several fixed networks in series with variable losses. The circuits are interconnected as shown in figures 3-4a and 3-5a. For the networks shown the characterizing functions are respectively given by equations 3-3a and 3-3b.

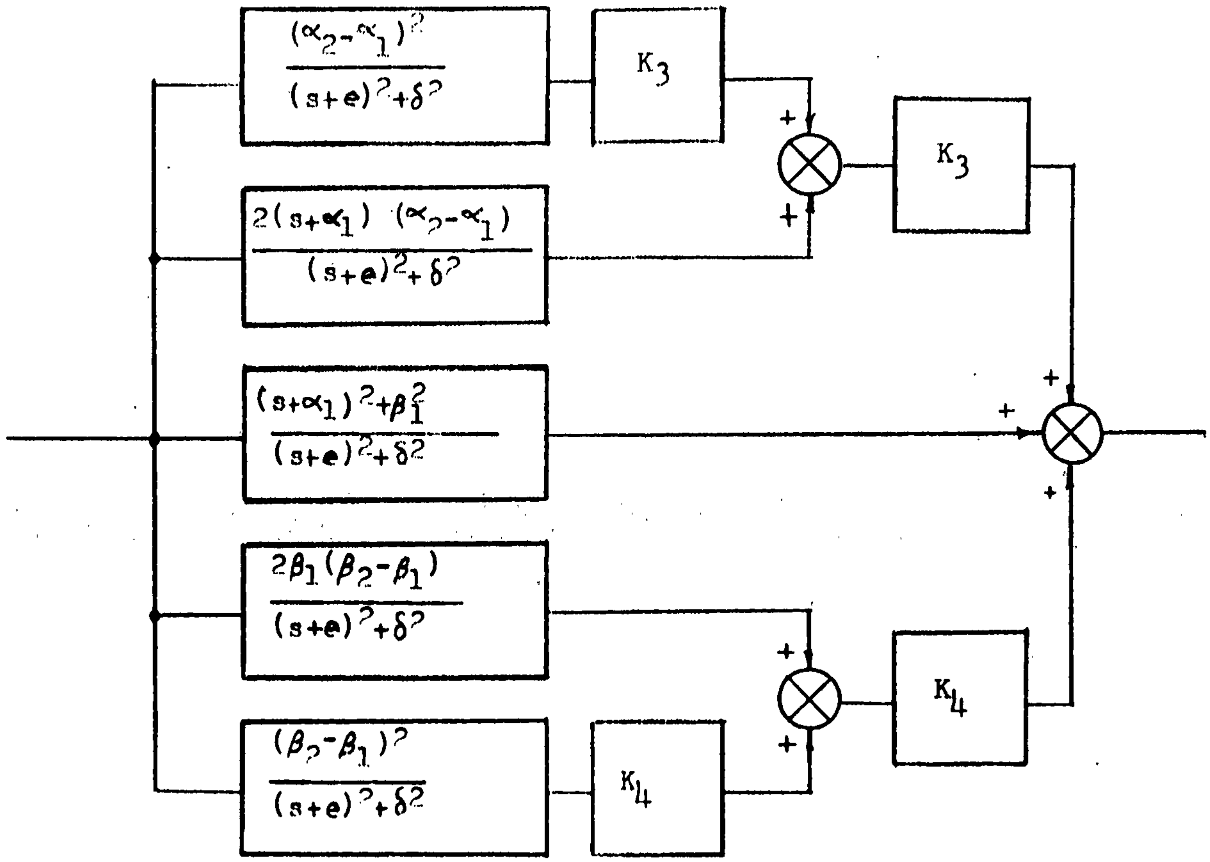


Figure 3-4a Compensator Element Containing a Pair of Variable Complex Zeros

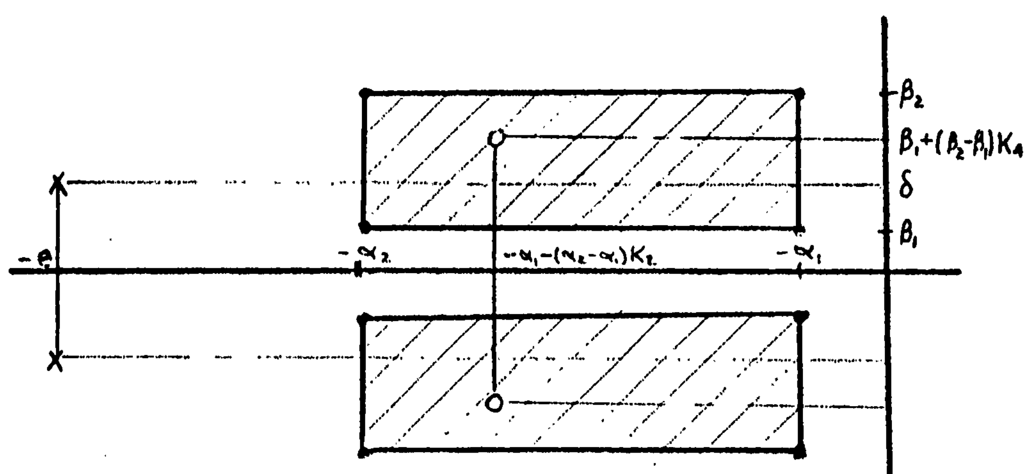


Figure 3-4b Root Locus Plot for Variable Zeros

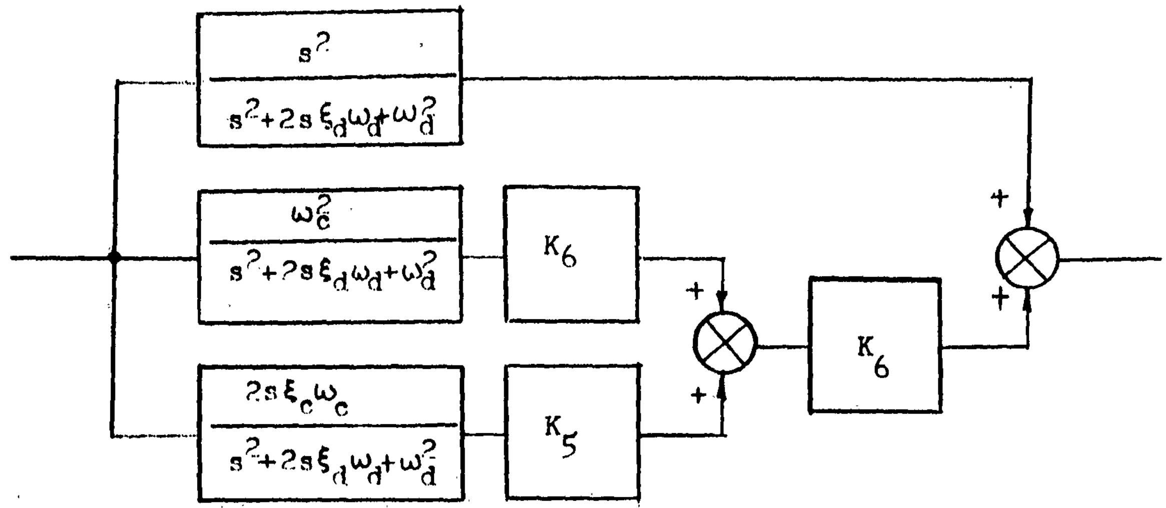


Figure 3-5a Alternate Compensator Element for a Pair of Variable Complex Zeros

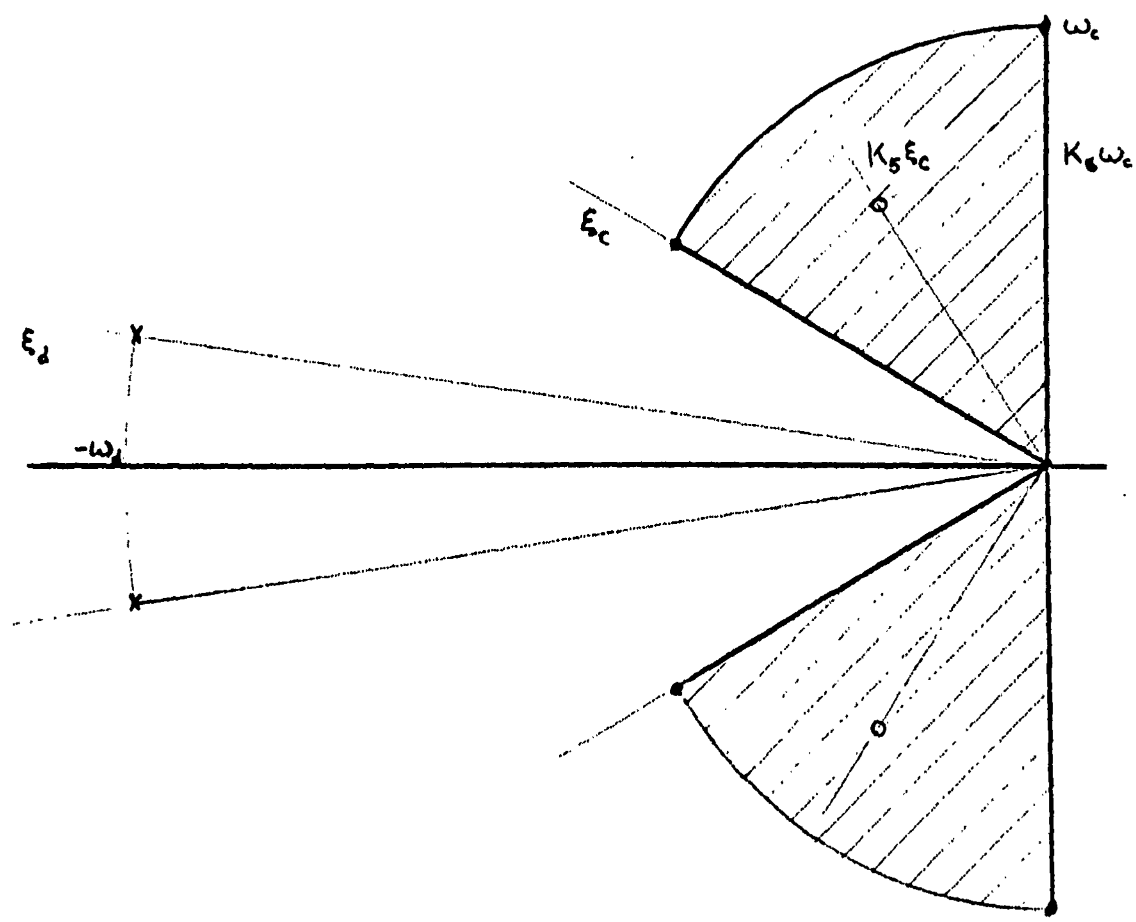


Figure 3-5b Root Locus for the Alternate Compensator

$$G_2(s) = \frac{[s+\alpha_1+(\alpha_2-\alpha_1)K_3(t)]^2 + [\beta_1+(\beta_2-\beta_1)K_4(t)]^2}{(s+e)^2 + \delta^2} \tag{3-3a}$$

$$G_3(s) = \frac{s^2 + 2s[\xi_c K_5(t)][\omega_c K_6(t)] + [\omega_c K_6(t)]^2}{s^2 + 2s\xi_d\omega_d + \omega_d^2} \tag{3-3b}$$

A plot showing the respective regions in which the zeros are located is shown in figures 3-4b and 3-5b. The positions of these zeros in the regions are determined by the values of the two respective losses. A convenient method of varying these loss values is thru the use of potentiometers; in both cases these potentiometers are positioned by two small motors.

3.5 Variable Poles

The compensation of a process that contains variable zeros requires the use of compensator elements that contains variable poles.[1] The technique of linear combination employed in constructing compensating elements that contains variable zeros is not applicable to this problem. A modified

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[1]For subsequent reference the variable poles of the process characterizing function are referred to as the variable poles of the process, etc.

approach is used: the construction of a compensator element that contains a variable pole (pair of complex poles) is accomplished by placing a compensator element that contains a variable zero (pair of complex zeros) in the feedback path of a high gain amplifier. The region over which the zeros in the feedback path vary is then the region over which the compensator element poles vary; the position of the poles is the same as the position of the zeros.

3.6 Conclusion

Any compensator characterizing function, that is rational in s, is realizable using the technique presented in this chapter; it is necessary however, that the poles of this function remain in the left hand s-plane. For such a compensator a tandem combination of some of the five basic compensating elements is required. Each of these elements contains a combination of fixed networks, fixed gains and variable losses. It is proposed that these losses be obtained by using potentiometers that are positioned by control motors. For such a system several potentiometers are required, but only one control motor is required per degree of freedom of the compensator. As a result, the compensator is general, easy to design, simple to construct, and practical to adjust. The criterion used to adjust the compensator is presented in the next chapter.

CHAPTER 4

Compensator Adjustment

Design of the compensator is accomplished on the basis that all the fixed parameters of the compensator characterizing function are known. These are determinable by measurements made upon the process before control is attempted. There remain then only the variable parameters: some are known functions of time and some are unknown functions of time. The known functions of time are fed to the compensator from programmed generators; these offer little difficulty. The unknown functions of time are generated by the adaptive circuitry on the basis of measurements; these constitute the major problem. As a result, consideration is given only to those parameters which are unknown functions of time. A measurement scheme is proposed for generating the unknown parameter control functions and an effort is made to construct a system that allows for analysis. A discussion of the errors involved is presented.

4.1 General

Adjustment of the variable parameters of the compensator is based upon measurements made on the compensated process. To do this involves the generating of several frequency weighted signals by passing the input and output signals thru similar families of filters. The power in these signals are then time weighted and a comparison of the energy in pairs of signals is used to generate error signals. These error signals are then used to correct the errors that exist in the compensated process. As a result of this effort, approximations to the unknown parameter control functions are generated. In general a more slowly varying process allows for more accurately approximated control functions.

Selection of the filters and the time weighting functions are important in determining the behavior of the compensator. An effort is made in selecting the filters to have each of the resulting signals particularly sensitive to only one variable parameter. Failure to insure this introduces coupling between correcting loops and thus makes possible multi-loop oscillations. Selection of the time weighting function is made to achieve as great a measurement accuracy as possible with the process variations involved. Choosing the time weighting function incorrectly leads to either an inaccurate or an insensitive system.

4.2 Principle of Operation

The principle of operation of the proposed system involes maintaining the magnitude of the transfer function of the compensated process constant by checking its energy transfer characteristic. To do this one of two approaches is used: Either the output is passed thru a transfer function which is the reciprocal of the desired control system transfer function[1] or the input is passed thru a model for which the transfer function is the one desired for the control system. Figures 4-1a and 4-1b show examples of the first and second approaches respectively. In the first method a signal denoted as $r'(t)$ is generated; in the second method the signal generated is denoted as $c'(t)$. In either case, the two signals [e.g., $r(t)$ and $r'(t)$ or $c(t)$ and $c'(t)$] are used to generate several filtered signals; one signal is generated for each variable (i.e., α, β, K_c, ω_o, ξ, etc.).

From this point on both methods are identical. A root mean square value of the time weighted history of each of the signals is generated and these values are compared for pairs of filtered signals. If a difference is found for a given pair (e.g., an error signal), it is used to vary the parameter associated with that pair in an effort to reduce the difference to zero.

For a perfectly compensated system the signal pair are identical for any arbitrary input and the errors are therefore all zero. Conversely, if the errors are all zero, for any arbitrary input, the system is perfectly compensated. This condition is a result of the choice of one error signal per variable parameter. The proposed system is now mathematically formulated.

[1]At least approximately over the band of interest.

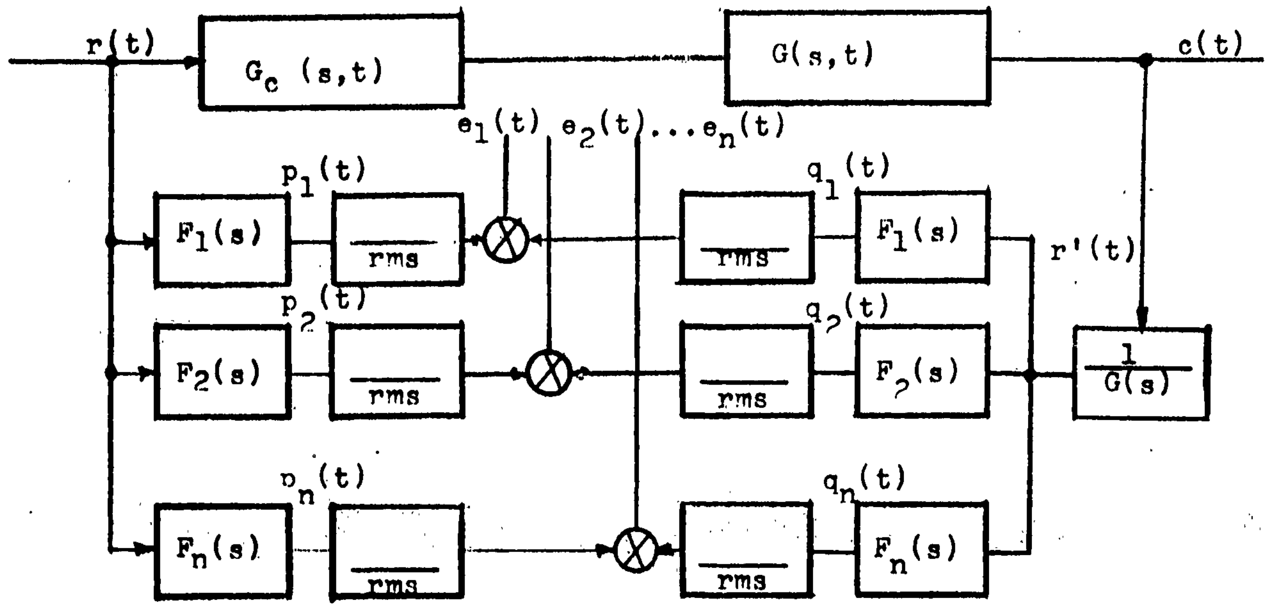


Figure 4-1a System Block Diagram

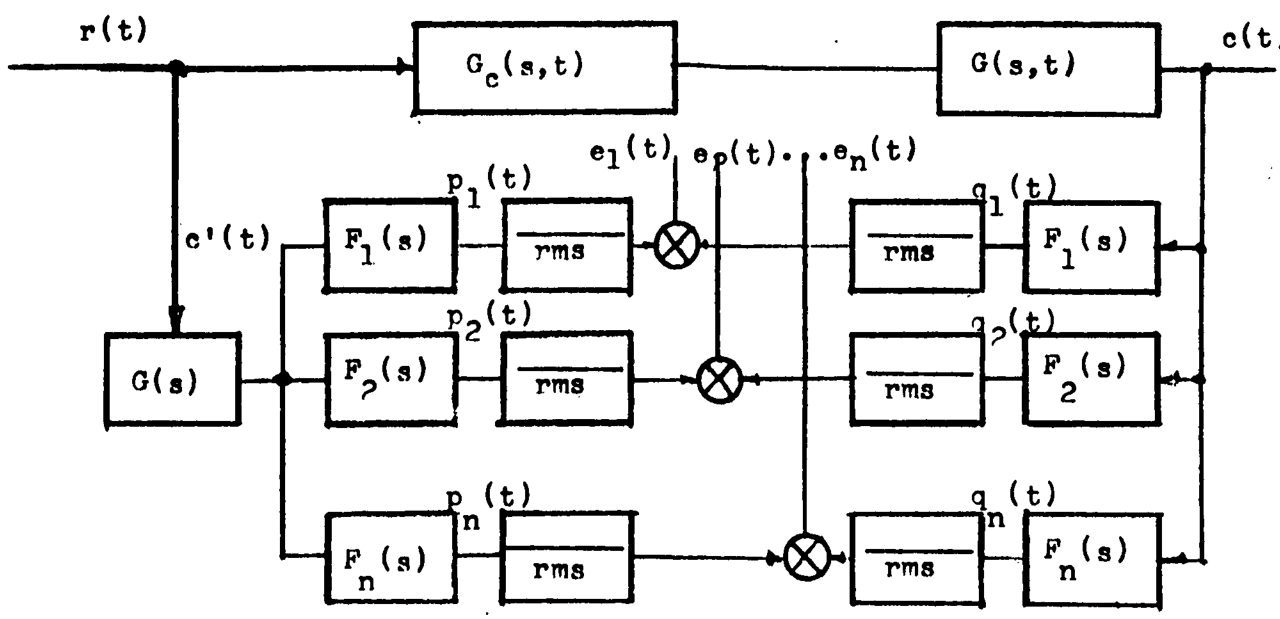


Figure 4-1b Alternate System Block Diagram

4.3 A Mathematical Model

Both system configurations are redrawn to exhibit the operation of the i^{th} loop for the time instant t_o. These are shown in Figures 4-2a and 4-2b where it is seen that with the exception of signal prefiltering both systems operate identically. Each of the signals shown [i.e., $p_i(t)$, $q_i(t)$, etc.] is then divided into three parts - the first part is denoted by a second subscript of one [i.e., $p_{i1}(t)$, $q_{i1}(t)$, etc.] and is that portion of the signal occurring prior to the time t_o-T + 5t_r; the second part is denoted by a second subscript of two and is that portion of the signal occurring between the time t_o-T + 5t_r and the time t_o; the third part is denoted by a second subscript of three and is that portion of the signal occurring after the time t_o. A comparison between the root mean square values of $p_{i2}(t)$ and $q_{i2}(t)$ is then made and the difference, which is considered to be due to an adjustment error, is used to drive a compensator adjusting control motor. This corresponds to using the rectangular time weighting function shown in Figure 4-3 and assuming that $q_{i2}(t)$ is the total output for an input of $p_{i2}(t)$. This second assumption neglects so-called "end effects" and is reasonable for those cases where T is much greater than t_r. A discussion of the approximation error involved is included subsequently.

For the systems being considered the error signal is denoted by $e_i(t_o)$ and T-5t_r is denoted by T_o giving

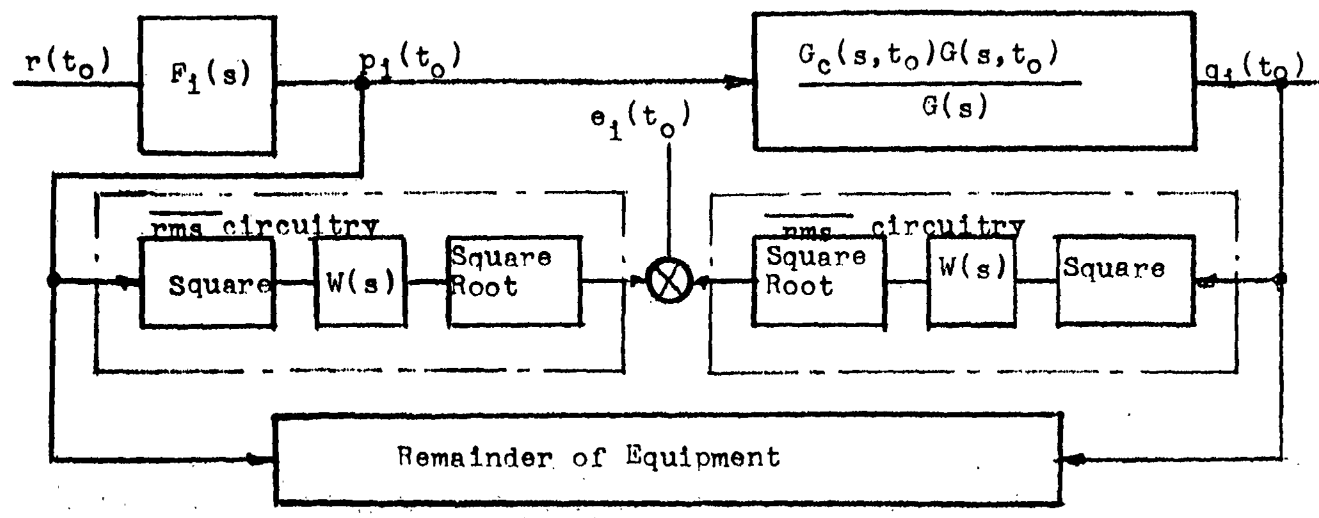


Figure 4-2a Model for System

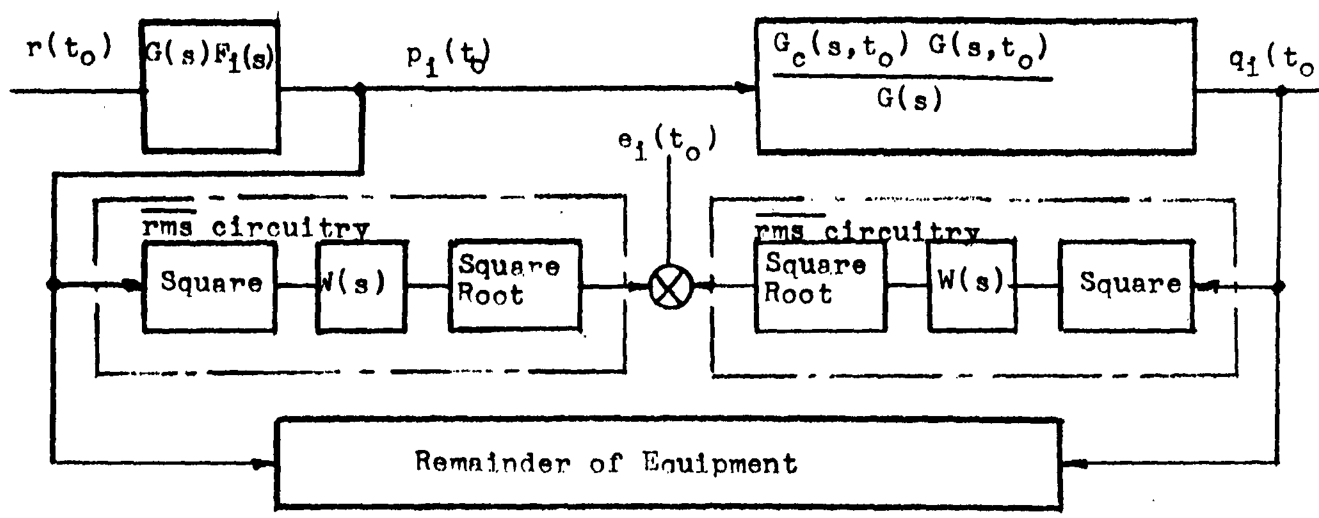


Figure 4-2b Model for Alternate System

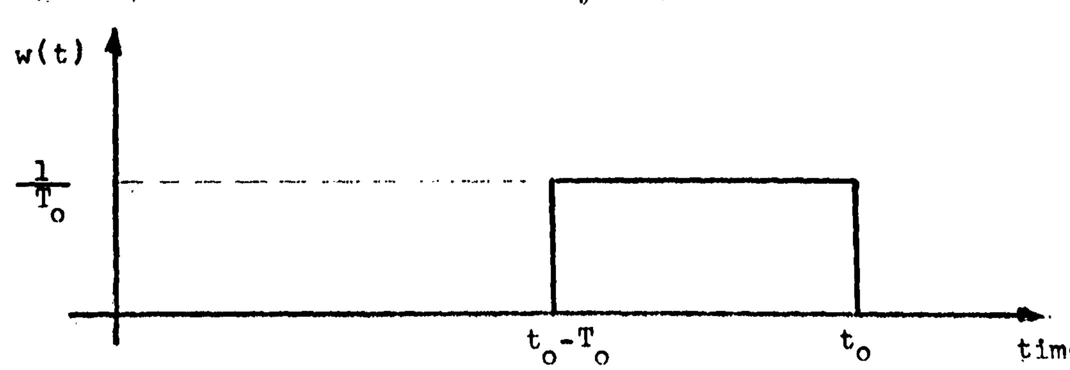


Figure 4-3 Rectangular Weighting Function

$$e_i(t_o) = \sqrt{\frac{1}{T_o}\int_{-\infty}^{\infty} p_{12}^2(t)dt} - \sqrt{\frac{1}{T_o}\int_{-\infty}^{\infty} q_{12}^2(t)dt} \qquad (4\text{-}1)$$

This is rewritten, using Parsevals Theorem and the approximation that $q_{12}(t)$ is the output due to $p_{12}(t)$, in equation 4-2.

$$e_i(t_o) = \sqrt{\frac{1}{2\pi T_o}\int_{-\infty}^{\infty} \left| P_{12}(\omega) \right|^2 d\omega} - \sqrt{\frac{1}{2\pi T_o}\int_{-\infty}^{\infty} \left| Q_{12}(\omega) \right|^2 d\omega}$$

$$= \sqrt{\frac{1}{2\pi T_o}\int_{-\infty}^{\infty} \left| P_{12}(\omega) \right|^2 d\omega}$$

$$- \sqrt{\frac{1}{2\pi T_o}\int_{-\infty}^{\infty} \left| P_{12}(\omega) \right|^2 \frac{\left| G_c(t_o,\omega) \right|^2 \left| G(t_o,\omega) \right|^2}{\left| G(\omega) \right|^2} d\omega}$$

(4-2)

The choice of $F_i(\omega)$ is now constrained so as to insure that an increase or decrease in the i^{th} parameter causes either an increase or decrease in the term

$$\left| \frac{G_c(t_o,\omega)\; G(t_o,\omega)}{G(\omega)} \right| ,$$

for all ω for which $P_{12}(\omega) \neq 0$; thus the sign of $e_i(t_o)$ always indicates the direction in which the i^{th} compensator parameter must be varied. Proper application of this signal to the corresponding control motor then insures that corrective compensator adjustments are made as required. In addition, defining

$$\overline{P}_i(t_o) = \sqrt{\frac{1}{2\pi T_o} \int_{-\infty}^{\infty} \left| P_{12}(\omega) \right|^2 d\omega}$$

and passing $e_i(t)$ thru an AGC amplifier with gain $D/\overline{P}_i(t_o)$ yields:[2]

$$\dot{K}_i(t_o) = \frac{D}{\overline{P}_i(t_o)} e_i(t_o)$$

$$= D\left[1 - \frac{1}{\overline{P}_i(t_o)} \sqrt{\frac{1}{2\pi(t_o)} \int_{-\infty}^{\infty} \left| P_{12}(\omega) \right|^2 \frac{\left| G_c(t_o,\omega)G(t_o\omega) \right|^2 d\omega}{\left| G(\omega) \right|^2}}\right]$$

$$= D\,[1-G_i(t_o)] \qquad (4\text{-}3)$$

---

[2]For physical systems $\overline{P}_i(t_o)$ is limited below by noise and above by the saturation of the previous equipment.

With $\dot{K}_i(t_o)$ applied to the control motor a signal $K_i(t_o)$ is generated which represents the approximation to the i^{th} compensator parameter control function. The accuracy of the approximation depends on the adaptive loop.

4.4 Discussion of the Approximation Errors

In the investigation of the error involved in the approximation of the energy in $q_{12}(t)$, $q_1(t)$ is divided into two parts: the first is denoted by $q_1'(t)$ and is that portion of $q_1(t)$ caused by $p_{11}(t)$; the second is denoted by $q_1''(t)$ and is that portion of $q_1(t)$ caused by $p_{12}(t)$. Each of these functions is then subdivided into three time intervals which are denoted by the use of second subscripts, as in the previous section. The percentage approximation error $e_{ai}(t)$ is then given by equation 4-4. Applying Parsevals Theorem, substituting the terms above and simplifying them gives equation 4-5.

$$e_{ai}(t) = \frac{\frac{1}{2\pi T_o}\int_{-\infty}^{\infty}\left|Q_{12}(\omega)\right|^2 d\omega - \frac{1}{2\pi T_o}\int_{-\infty}^{\infty}\left|Q_1''(\omega)\right|^2 d\omega}{\frac{1}{2\pi T_o}\int_{-\infty}^{\infty}\left|Q_1''(\omega)\right|^2 d\omega} \tag{4-4}$$

$$e_{ai}(t) = \frac{2\int_{t_o-T_o}^{t_o} q'_{i2}(t)q''_{i2}(t)dt + \int_{t_o-T_o}^{t_o} \left[q'_{i2}(t)\right]^2 dt - \int_{t_o}^{\infty} \left[q''_{i3}(t)\right]^2 dt}{\int_{t_o-T_o}^{t_o} \left[q''_{i2}(t)\right]^2 dt + \int_{t_o}^{\infty} \left[q''_{i3}(t)\right]^2 dt} \tag{4-5}$$

The expression for $e_{ai}(t)$ in equation 4-5 is now interpreted. Since T_o is much greater than the largest process response time constant all of the integrals in equation 4-5 except the first integral in the denominator are approximately independent of T_o. As a result, for a properly chosen T_o(e.g., sufficiently large) the error $e_{ai}(t)$ is negligibly small.

The third integral in the numerator and the second integral in the denominator represent the energy output resulting from the energy stored in the mathematical model at time t_o (see Figure 4-2a or 4-2b). The second integral in the numerator represents the energy in the output signal caused by the energy stored in the mathematical model at time t_o-T_o; the first integral in the numerator depends directly upon this signal. For a perfectly compensated process the mathematical model has a transfer function of 1 and hence these first four integrals are all equal to zero. The last remaining integral is the energy output during the interval between $t_o - T_o$ and t_o, that is caused by the input $p_{12}(t)$. This integral is exactly equal to the input energy, during the interval between $t_o - T$ and t_o, if the process is perfectly compensated. As a result, the approximation error is zero for a perfectly compensated process and negligibly small for a properly compensated process.

4.5 Alternative Weighting Function

The rectangular weighting function discussed above corresponds to selecting the filter block W(s), in the $\overline{rms}$ circuitry shown in Figure 4-2a and 4-2b, to have a transfer function of

$$e^{-\frac{sT_o}{2}} \frac{\sinh \frac{sT_o}{2}}{\frac{sT_o}{2}} \, .$$

From practical considerations this transfer function is approximated by $\frac{3/T_o}{s + 3/T_o}$, which corresponds to an exponential time weighting function. The area under the two weighting functions are equal and a graph comparing the two functions is shown in Figure 4-4.

The operation of the system with the exponential weighting function is the same as the operation of the system with the rectangular weighting function. The functions $p_{11}(t)$, $p_{12}(t)$, $q_{11}(t)$ and $q_{12}(t)$ are redefined to reflect this modification

$$\left[\text{i.e., } p_{12}(t) = \sqrt{\frac{3}{T_o}}\, e^{\frac{3(t-t_o)}{2T_o}}\, p_1(t)\, U(t_o - t) \text{ , etc.} \right]$$

and an analysis similar to that presented in section 4.3 is applied. The result is again equation 4-3, but the justification for the approximations involved is slightly modified.

4.6 Conclusion

The measurement scheme presented for generating the control functions necessary in adjusting the parameters of the tandem compensator is based on an energy transfer criterion. The interval of time over which such a measurement is made is of necessity much larger than the response time constants of the compensated process involved; this de-emphasizes end effects. In addition, the measurement time interval is of necessity small with respect to the time required for significant process variation; this insures a sensitive, properly adapting system. The time constant of the proposed exponential weighting function is therefore a compromise between these two requirements. As a result, there is an "end effect error" introduced. This error is a part of the measurement error expected in adjusting the compensator.

The balance of the measurement error results from the choice made in generating the control function by integrating the error signal. This part of the measurement error

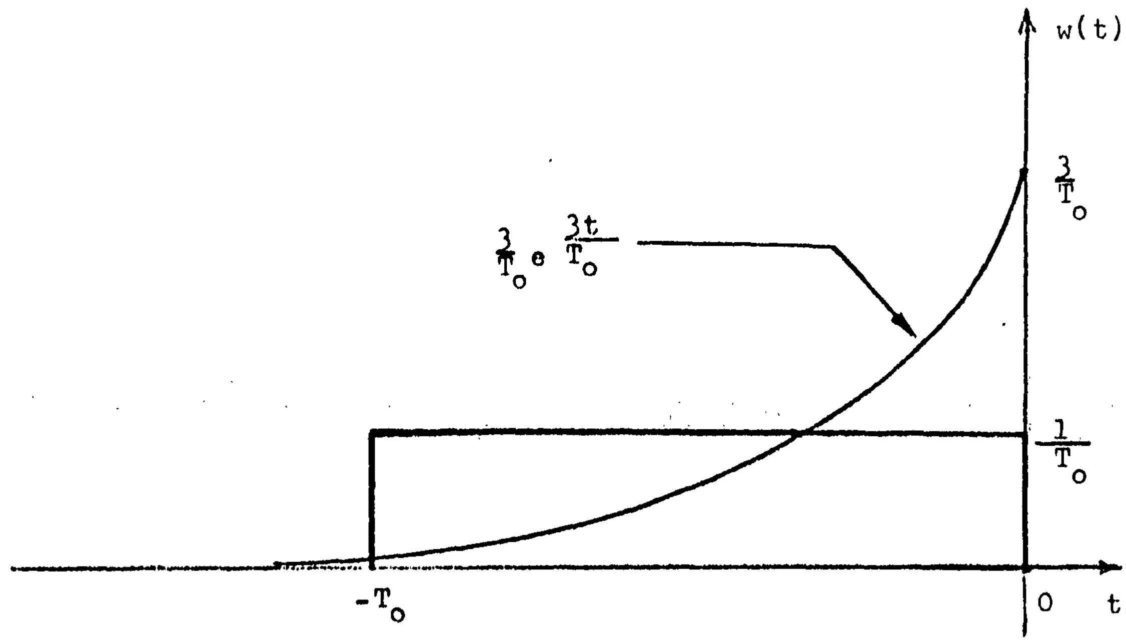


Figure 4-4 Alternate Weighting Function

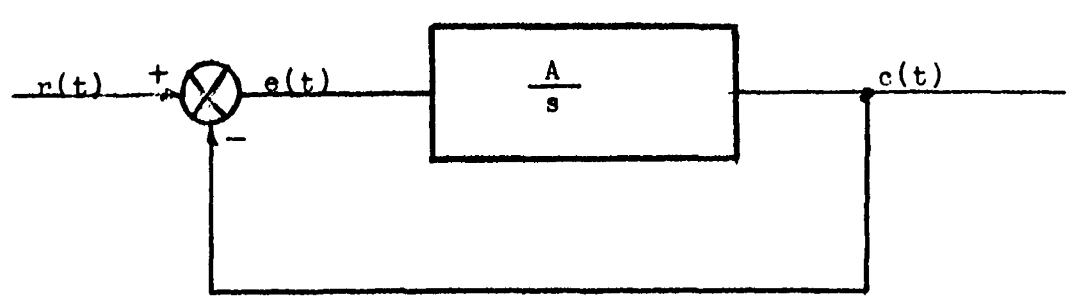


Figure 4-5 Simple Feedback System

is similar to the error that exists in linear systems when a signal is applied to an integrator having a unity-gain feedback loop (see Figure 4-5). The system does, however, continually try to maintain a fixed appearance by driving the compensator parameters so as to track the process parameters and thereby cause cancellation. The operation of some example adapted processes is investigated next.

Chapter 5

Processes With One Variable Coefficient

The previous chapters are devoted to the development and presentation of a general, simple, practical design technique for use in the solution of variable process control problems. Throughout these sections an effort is made to create a system that allows for analysis. It is thus appropriate at this point to investigate the application of this technique to several different processes. This chapter is devoted to such investigation for those processes which contain only one variable coefficient in their process characterizing function. A process that exhibits a variable gain is considered first. A process that contains a variable real pole and a process that contains a variable real zero are then considered and compared. Several examples are included among which is a problem that appears impossible to solve using linear feedback theory. The problem of controlling a process that contains a pair of variable complex poles or zeros, for which only one parameter varies, (i.e. α, β, ξ, or ω_n) is deferred to the next chapter since it is a special case of a more general problem.

5.1A Process With Variable Gain

A process that contains a variable gain term in its characterizing function is compensated by the use of a single tandem variable gain compensator and a single adaptive loop, as shown in figure 5-1a. For this process, the characterizing function is

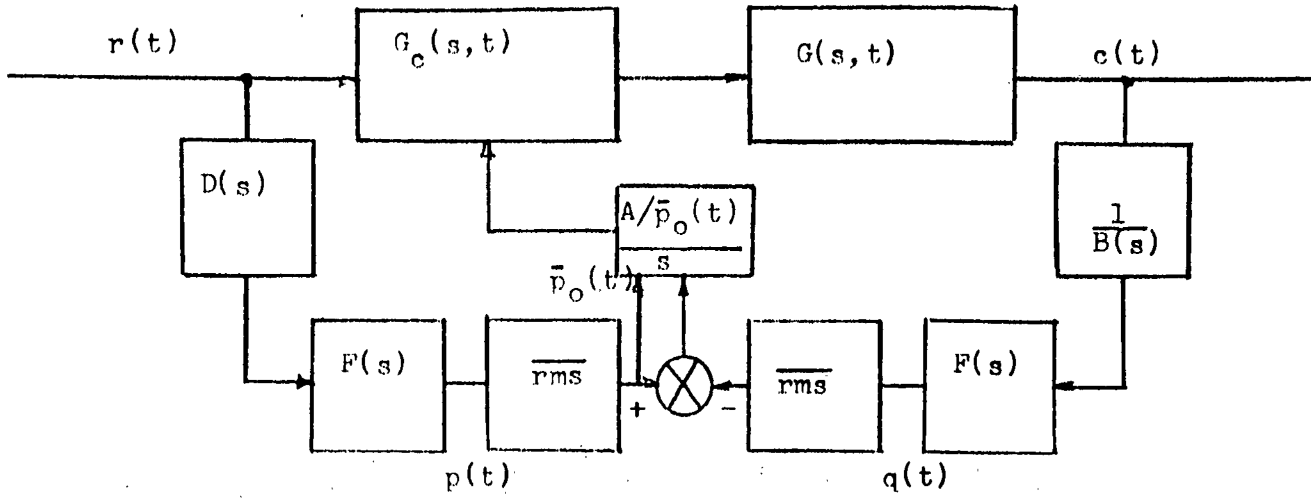
$$G\ (s,t) = K_m\ G_1(s)/K_o(t) \qquad (5\text{-}1)$$

The assumption is made that all of the desired signal shaping networks are included with the process and thus when the variable parameter is fixed at its nominal value a satisfactory control system is obtained (this assumption is made for all systems subsequently considered). As a result, a model for the operation of the adaptive loop is shown in figure 5-1b. The signal filtering block that generates p(t) given r(t) is labeled H(s); it is equal to either F(s) or F(s) $G_m(s)$ and depends upon whether a system that uses r'(t) or one that uses c'(t) is selected. Since this does not directly effect the operation of the system H(s) is neglected from further consideration. System operation is investigated in terms of p(t).

In equation (5-1) K_m is the minimum value of the process gain and hence $K_o(t)$ varies between zero and one. The compensator as a result is chosen to have the characterizing function

$$G_c(s,t) = K_n K_c(t)/K_m \qquad (5\text{-}2)$$

Since K_n is the desired process gain, $\breve{K}_c(t)$ also varies between zero and one. To compensate the process $K_c(t)$ must be equal to $K_o(t)$ and thus the adaptive loop operates to cause the tracking of $K_o(t)$ by $K_c(t)$ in an effort to achieve this equality.



*Use D(s)= G(s) and B(s)= 1,
B(s)= G(s) and D(s)= 1, or any combination such that
D(s)·B(s)= G(s)

Figure 5-1a Variable Gain System

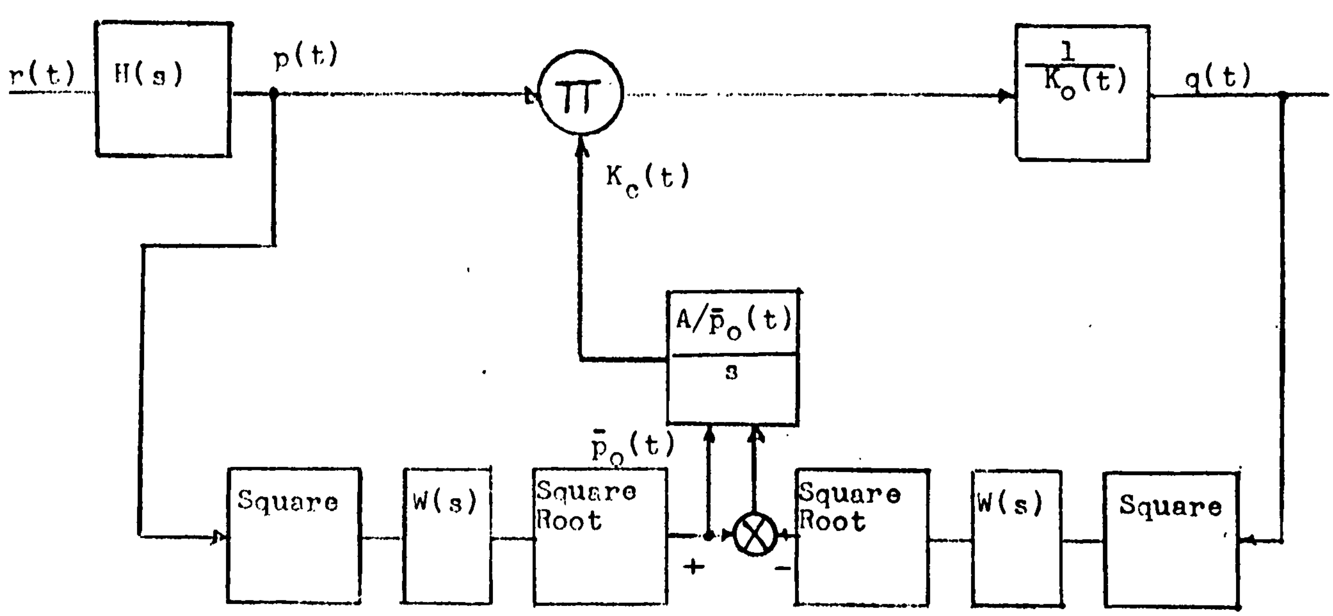


Figure 5-1b Model for Variable Gain System

For the system being considered equation (4-3)

$$\dot{K}_c(t) = A\,[1 - G_1(t_o)] \tag{4-3}$$

is rewritten as

$$\dot{K}_c(t) = A\,[1 - K_c(t)/K_o(t)\,] \tag{5-3}$$

since $G_1(t) = K_c(t_o)/K_o(t_o)$. The regrouping of terms gives the first order linear differential equation

$$\dot{K}_c(t) + [A/K_o(t)]\,K_c(t) = A \tag{5-4}$$

The general solution to this equation is

$$K_c(t) = e^{-\int_{-\infty}^{t} \frac{Adt}{K_o(t)}} \int_{-\infty}^{t} A\,e^{\int_{-\infty}^{t} \frac{Adt}{K_o(t)}}\,dt \tag{5-5}$$

Using equation (5-5) the output of the adaptive process is then

$$C(t) = [r(t)\;K_nK_c(t)/K_o(t)]*\,\mathcal{L}^{-1}[G(s)] \tag{5-6}$$

where $\mathcal{L}^{-1}\,[G(s)]$ is the impulse response of the desired control system.

Thus the operation of the adaptive process is completely described; use, however, is primarily made of equation (5-5) in the investigation of the operation of the system since this equation indicates how well the adaptive circuitry is functioning. Several examples are presented in section 5.4.

5.2 A Process With A Variable Real Pole

A process that contains a variable real pole in its characterizing function is compensated by the system shown in figure 5-2a. The process is assumed to contain all the required fixed shaping circuitry and for a process characterized by

$$G(s,t) = \frac{G_1(s)}{[s + a\,K_o(t)]} \tag{5-7}$$

a tandem compensator that is characterized by

$$G_c(s,t) = \frac{[s + a\,K_c(t)]}{[s + a_o]} \tag{5-8}$$

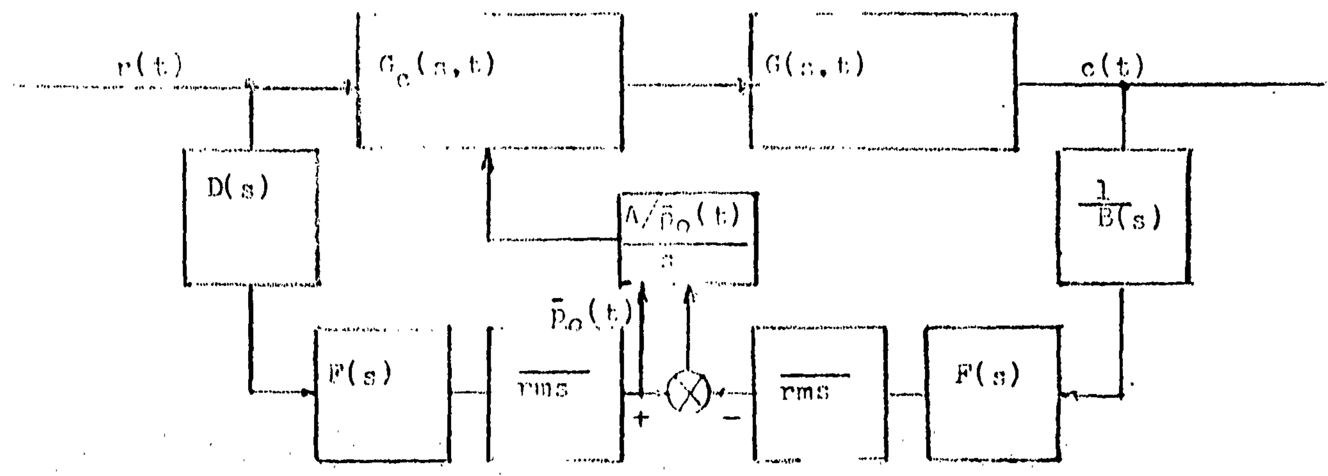
is used.

A model for this adaptive system is shown in figure 5-2b.

For "a" defined as the largest allowable value for the real pole, both $K_o(t)$ and $K_c(t)$ vary between zero and one. In addition, for a_m defined as the minimum value of the real pole and for $F(jw)$ a low pass filter with cutoff frequency well below $\omega = a_m$, $G_1(t_o)$ in equation (4-3) is approximated for the frequencies of interest by

$$G_1(t_o) \doteq \frac{1}{\overline{P}_c(t_o)} \sqrt{\frac{1}{2\pi T}\int_{-a_m}^{a_m} \left| P_{c2}(\omega) \right|^2 \left| \frac{\omega^2 + a^2K_c^2(t_o)}{\omega^2 + a^2K_o^2(t_o)} \right| d\omega}$$

$$\doteq K_c(t_o)/K_o(t_o) \tag{5-9b}$$



*Use D(s)= G(s) and B(s)= 1,
B(s)= G(s) and D(s)= 1, or any combination such that
B(s)D(s)= G(s).

Figure 5-2a Variable Pole System

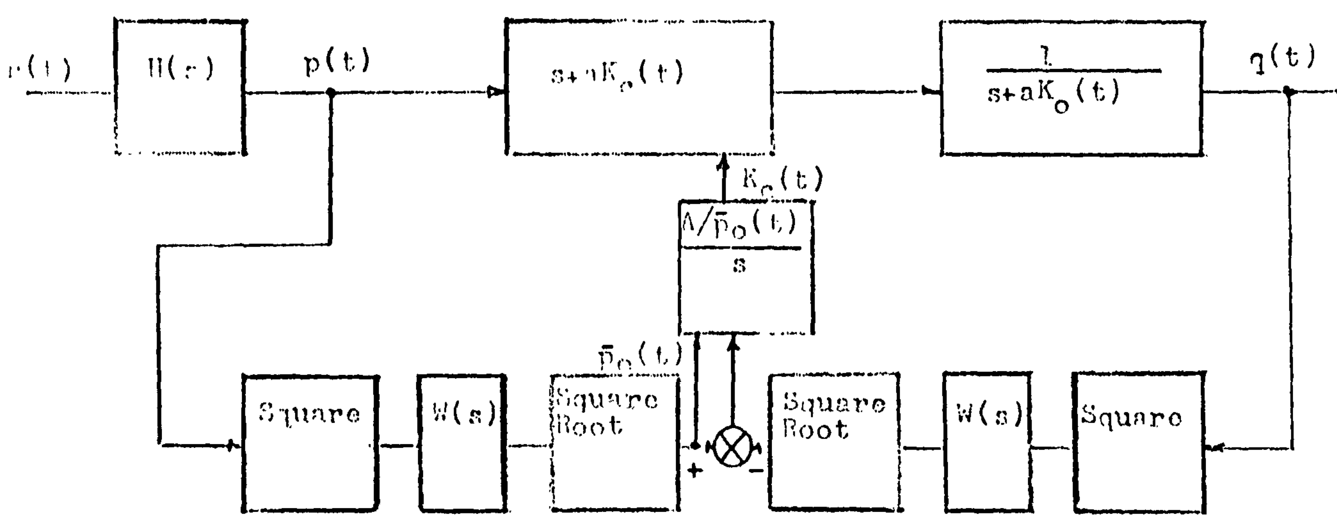


Figure 5-2b Model for Variable Pole System

Using this approximation, equation (4-3) is rewritten and solved; the solution is identical to the solution shown in equation (5-5). This equation allows for the analytic investigation of the operation of the compensator, for arbitrary pole variation, on the basis of low frequency gain.

For the case presented, the operation of the adaptive loop is similar to that of a variable gain adaptive loop; this is a result of the choice of a low pass filter for $F(s)$ and thus, the choice of an adaptive loop which is only sensitive to the "d.c. gain." Since the gain at any frequency is such as to insure tracking of $K_o(t)$ by $K_c(t)$[1], any choice for $F(s)$ is acceptable. Arbitrary choice of $F(s)$, however, does not lead to as simple a solution for $K_c(t)$ as does the choice considered.

5.3 A Process With A Variable Real Zero

A process that contains one variable real zero in its characterizing function is similar to a process that contains one variable real pole. The characterizing function for this process is

$$G(s,t) = [s + b\ K_o(t)]\ G_2(s) \tag{5-10}$$

For a tandem compensator, a network with a characterizing function of

$$G_c(s,t) = \frac{[s + b_o]}{[s + b\ K_c(t)]} \tag{5-11}$$

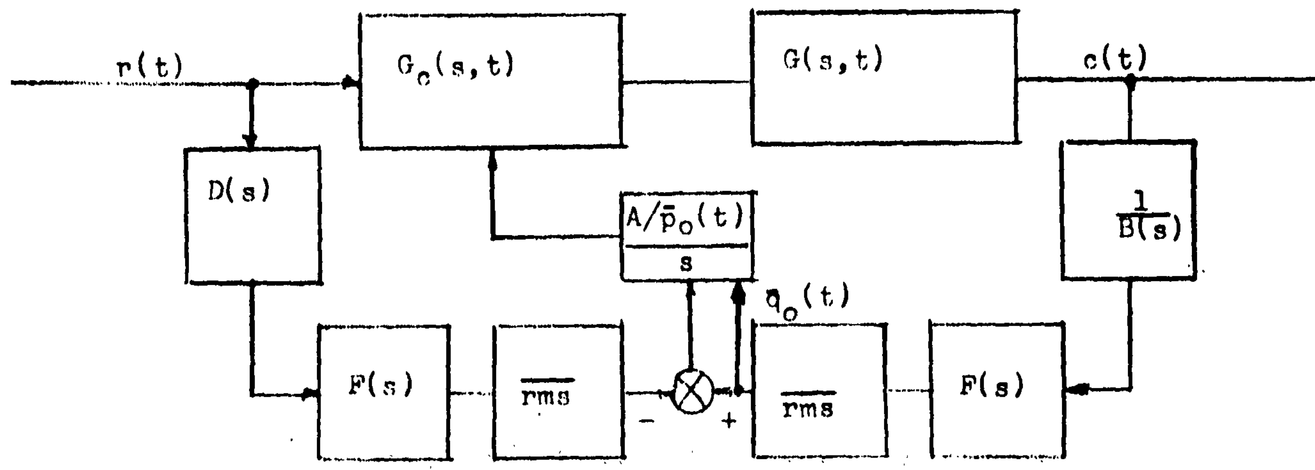
[1]This can simply be seen from a pole-zero diagram for the model.

is used. An adaptive system for this process is shown in figure 5-3a and a model for the operation of this adaptive loop is shown in figure 5-3b. The similarity between this system and the variable pole system is obvious from comparison of the models.

The operation of the adaptive loop with F(s) restricted as in the variable pole case is also given by equation, (5-5). Since the gain at any frequency is such as to insure tracking of $K_o(t)$ by $K_c(t)$, any choice for F(s) is again acceptable. The similarity in design between variable pole and variable zero systems thus indicates that only one of the two need be investigated. Subsequent discussions are, therefore, restricted to variable pole terms; a variable zero term is handled in a manner analogous to that used to handle the corresponding variable pole term.

5.4 Examples

In each of the three systems considered the variable parameter is normalized so as to reduce the problem to one involving a coefficient that varies between zero and one. This coefficient corresponds exactly to that variable gain in the compensator with which compensation is achieved. Under certain restrictions the behavior of the compensator variable gain term is governed by the same differential equation for all three systems; as a result all of the systems respond identically to a given type of process variation. This response is given by



*Use D(s)= G(s) and B(s)= 1,
B(s)= G(s) and D(s)= 1, or an combination such that
B(s)D(s)= G(s).

Figure 5-3a Variable Zero System

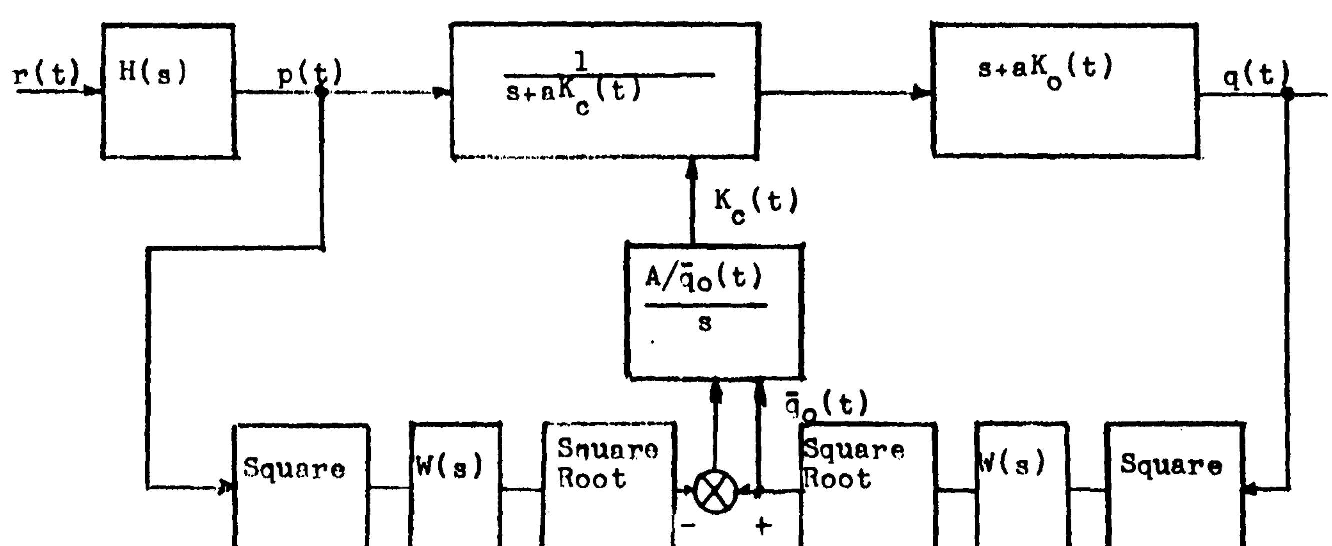


Figure 5-3b Model for Variable Zero System

$$K_c(t) = e^{-\int_{-\infty}^{t} \frac{Adt}{K_o(t)}} \int_{-\infty}^{t} A\, e^{\int_{-\infty}^{t} \frac{Adt}{K_o(t)}} dt \qquad (5\text{-}5)$$

For the three systems investigated two examples are considered. The first involves a step variation in the variable parameter; the second involves a parameter that starts at some nominal value and decreases with time. For the first problem $K_o(t)$ is given by

$$K_o(t) = K_{o1} + (K_{o2} - K_{o1})\, U_{-1}(t) \qquad (5\text{-}12)$$

This corresponds to a step variation from K_{o1} to K_{o2}. The response of $K_c(t)$ to this input is exponential and given by

$$K_c(t) = \begin{cases} K_{o1} & t < 0 \\ K_{o2} + (K_{o1} - K_{o2})\, e^{-\frac{At}{K_{o2}}} & t > 0 \end{cases} \qquad (5\text{-}13)$$

It is obtained by substituting equation (5-12) into equation (5-5) and shows that the time constant of the step response of the adaptive loop depends directly on the magnitude of the variable involved.

For the second problem $K_o(t)$ is given by

$$K_o(t) = K_{o1} / [1 + \alpha U_{-2}(t)] \qquad (5\text{-}14)$$

This corresponds to an inverse process gain that starts at a given value and then decreases with time. Substituting equation (5-14) into equation (5-5) and integrating gives, after some manipulation, a series expansion for $K_c(t)$

$$K_c(t) = \begin{cases} K_{o1} & t < 0 \\ K_{o1}\, e^{\frac{A}{\alpha K_{o1}}} - \sum_{n=0}^{\infty} \frac{\left(\frac{A}{\alpha}\right)^{n+1}\left(\frac{1}{2K_{o1}}\right)^{n}}{(2n+1)n!}\left[1 - (1-\alpha t)^2\right] e^{-\frac{A(1-\alpha t)^2}{2\alpha K_{o1}}} & t > 0 \end{cases} \tag{5-15}$$

Unfortunately, this expression does not give insight into the operation of the adaptive system. A family of curves is therefore presented which shows the behavior of $K_c(t)$ as a function of time with K_{o1} and α as parameters.

For the curves shown time is normalized with respect to the adaptive loop gain A and hence figure 5-4 shows $K_c(t)$ vs At for $K_{o1} = 1$ and $\alpha = 0.1A$, $0.5A$ and $1.0A$. Figure 5-5 shows the tracking errors for the above cases. It is noteworthy that for a larger rate of decrease in a parameter there is a larger maximum error but a faster convergence toward zero thereafter.

The operation of a variable gain process subject to the ramp variation considered above was simulated on a computer. The results of this investigation, a comparison with the results obtained above and a discussion of discrepancies is presented in the section on computer simulated systems.

5.5 A Problem That Appears Impossible to Solve

Prior to applying the design technique under consideration to more complex processes, it is worth-while to

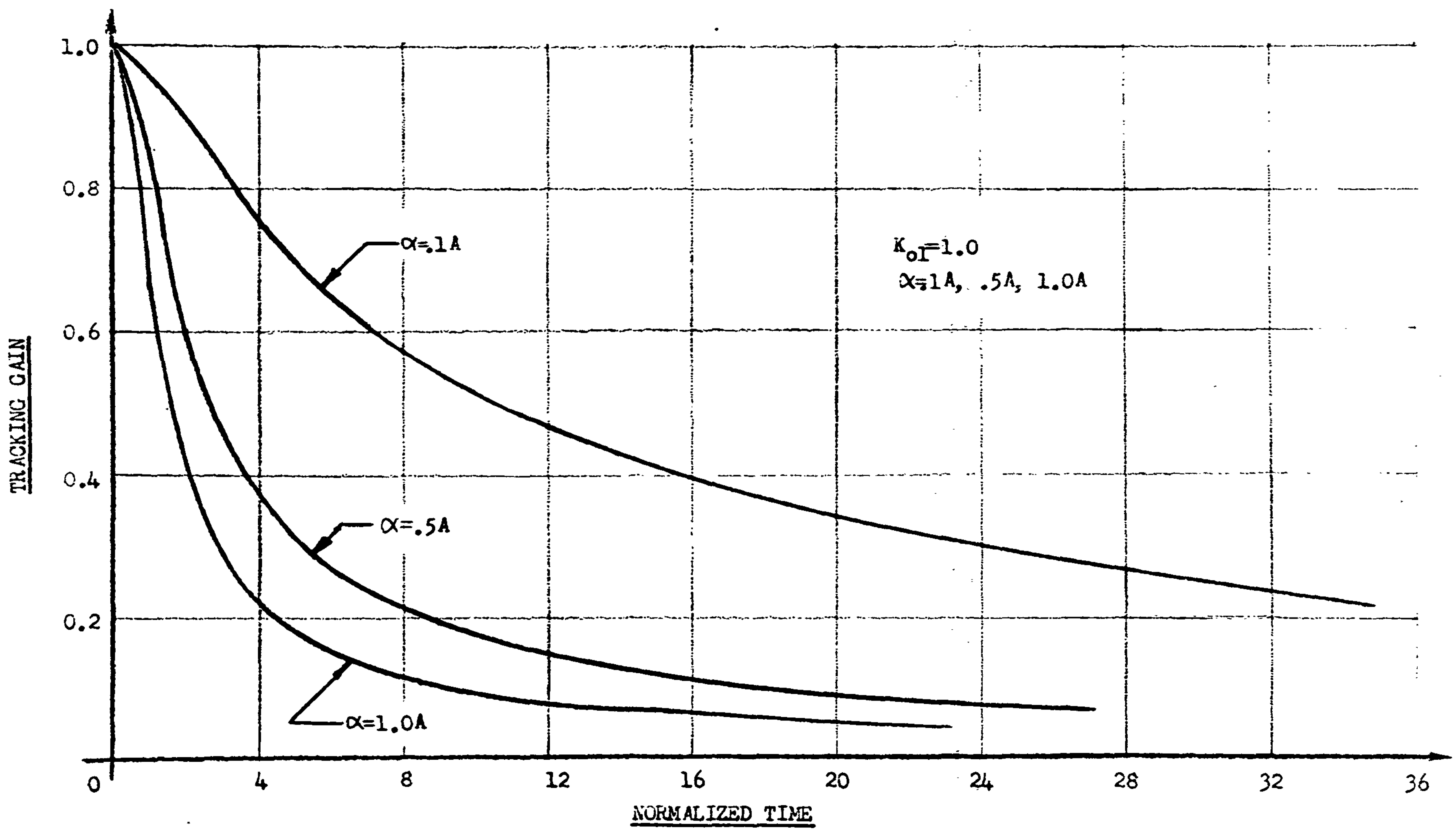


Figure 5.4 Tracking Gain vs. Normalized Time for Ramp Variation

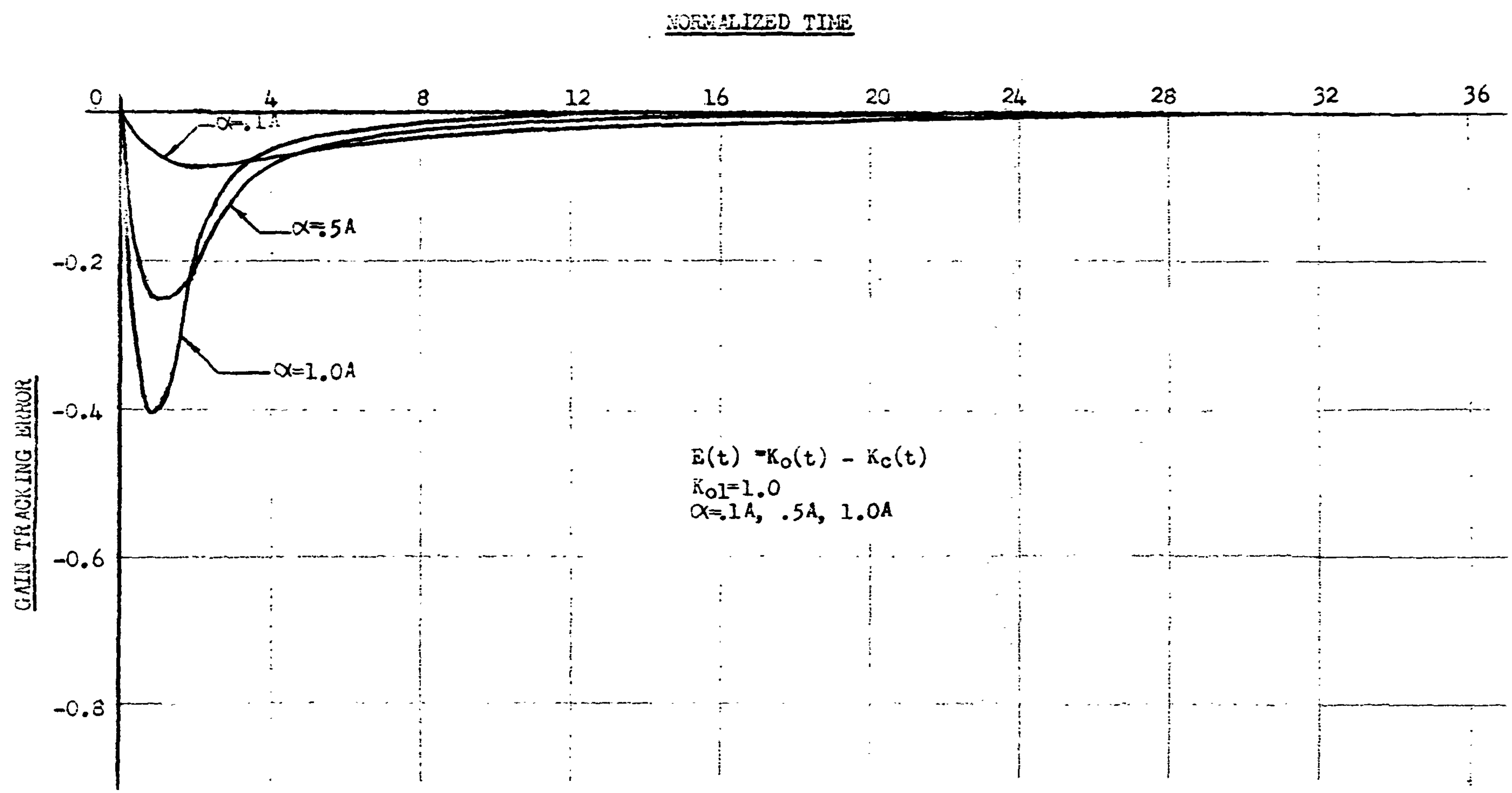


Figure 5.5 Gain Tracking Error vs. Normalized Time for Ramp Process Variation

demonstrate the usefulness of this approach. To do this, a problem that appears impossible to solve using linear feedback theory is selected and a simple practical solution is obtained. The problem is to design a control system for a variable gain process; the fixed part of the process is denoted by $G_p(s)$ and is given by

$$G_p(s) = \frac{4\sqrt{2}}{s + 0.4}\, e^{-\frac{s\pi}{16}} \tag{5-16}$$

The variable gain is denoted by $K_p(t)$ and is subject to change by fixed amounts at discrete intervals of time. This corresponds to a simple production process that is subject to loading and unloading at discrete times. The process gain is bounded as described by

$$1 \leq K_p(t) \leq 1000 \tag{5-17a}$$

and is subject to changes in accordance with

$$K_p(t) = 1 + \sum_{n=1}^{\infty} 0.1\ P(nT)\ U_{-1}(t-nT) \tag{5-17b}$$

where p(nT) is a discrete random variable, from an unknown distribution, that assumes the values -1, 0 and 1. The process is thus characterized by $K_p(t)\ G_p(s)$.

From production considerations, two requirements exist for any control system. These are:

1) The control system transfer characteristic must not vary (with time) in excess of 20% for any input signal frequency component below 0.4 radians per second.

2) The control system transfer characteristic bandwidth must remain as close to 0.4 radians per second as possible; The existance of high frequency noise prohibits the use of system bandwidth in excess of 16 radians per second.

For the above process, loop stability is a major problem when a control system is designed using linear feedback theory. The process gain variation of 60 db requires extensive shaping of the system open loop gain, to insure a stable system, since gain crossover varies over a frequency span of three decades. The possibility of the system bandwidth exceeding 800 radians per second therefore exists. As a result, a single loop system is not acceptable but a multi-loop approach, as suggested by Horowitz[2], is possible; for such a system the noise requirement is reformulated.

The possibility of an 800 radian per second system bandwidth in the multi-loop case remains. This is apparent when the system is redrawn to display the input-output feedback loop (such a loop exists for all multi-loop control system when the output is generated solely by a single-input single-output process).[3] As a result, a compensator network is required

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[2] I. M. Horowitz, "Design of Multiple-Loop Feedback Control Systems", I.R.E. Trans. on Automatic Control, Vol. AC-7, April, 1962.

[3] I. M. Horowitz, "Fundamental Theory of Automatic Linear Feedback Control Systems" I.R.E. Trans. on Automatic Control, Vol. AC-4 December, 1959.

to cancel the effect of the process delay term over the 800 radian per second band of interest. This corresponds to compensating for a maximum phase delay of 9000°; the required network is therefore unobtainable because of practical limitations and the problem appears impossible to solve.

Application of the design technique developed shows that this problem is identical to the first problem considered in section 5.4 under the condition that the adaptive loop settles between steps. Based on this assumption the characterizing function of the compensated process, subsequent to the occurance of a step in $K_p(t)$, is given by

$$T(s) = \left[1 - \left(1 - \frac{K_{o1}}{K_{o2}}\right) e^{-\frac{AT}{K_{o2}}}\right] G_p(s) \qquad (5\text{-}18)$$

It is obtained by using the results of equation (5-13).

The requirement for less than 20% variation in the gain of the process is immediately satisfied since the magnitude of $[1 - \frac{K_{o1}}{K_{o2}}]$ is bounded above by 0.1, for the allowable steps in process gain. The problem is then one of insuring that $(1 - \frac{K_{o1}}{K_{o2}})\ e^{\frac{-At}{K_{o2}}}$ is negligibly small for $t = T$ (i.e. less than ε_o); a choice of $A > \frac{5}{T}$ satisfies this requirement for $\varepsilon_o = 10^{-3}$. Hence for a typical problem and a time interval of one minute between changes in the process gain, an adaptive loop with a gain $A = 0.1$ is sufficient to give a satisfactory solution to the problem. A circuit corresponding to such a solution is shown in figure 5-6.

5.6 Conclusion

In each of the variable parameter problems investigated, care is taken in the selection of components to create a situation that permits analysis. In so doing all three adaptive systems are made to function in accordance with the same response equation. It is noted, however, that an alternate choice in adaptive loop circuitry is allowable. The square root circuits, for example, are included only for their value in simplifying the analysis; their omission may at times improve system operation. The choice made for $F(\omega)$ in the variable pole and variable zero problems is likewise intended to simplify analysis. Any alternate choice for $F(\omega)$ is allowable and some other filter may even improve system operation. The cost of making such changes is in general in the difficulty in analyzing the resulting system.

The results of the examples considered point out that in both cases the adaptive circuitry caused $K_c(t)$ to track $K_o(t)$ in such a manner as to reduce the tracking error toward zero. This indicates that the simple systems considered are stable for the types of process variations investigated. For some systems, however, the eventual decay of the error toward zero is not sufficient; a temporary error is intolerable. To remedy this problem a linear feedback loop is added around the adapted process; its function is to desensitize the

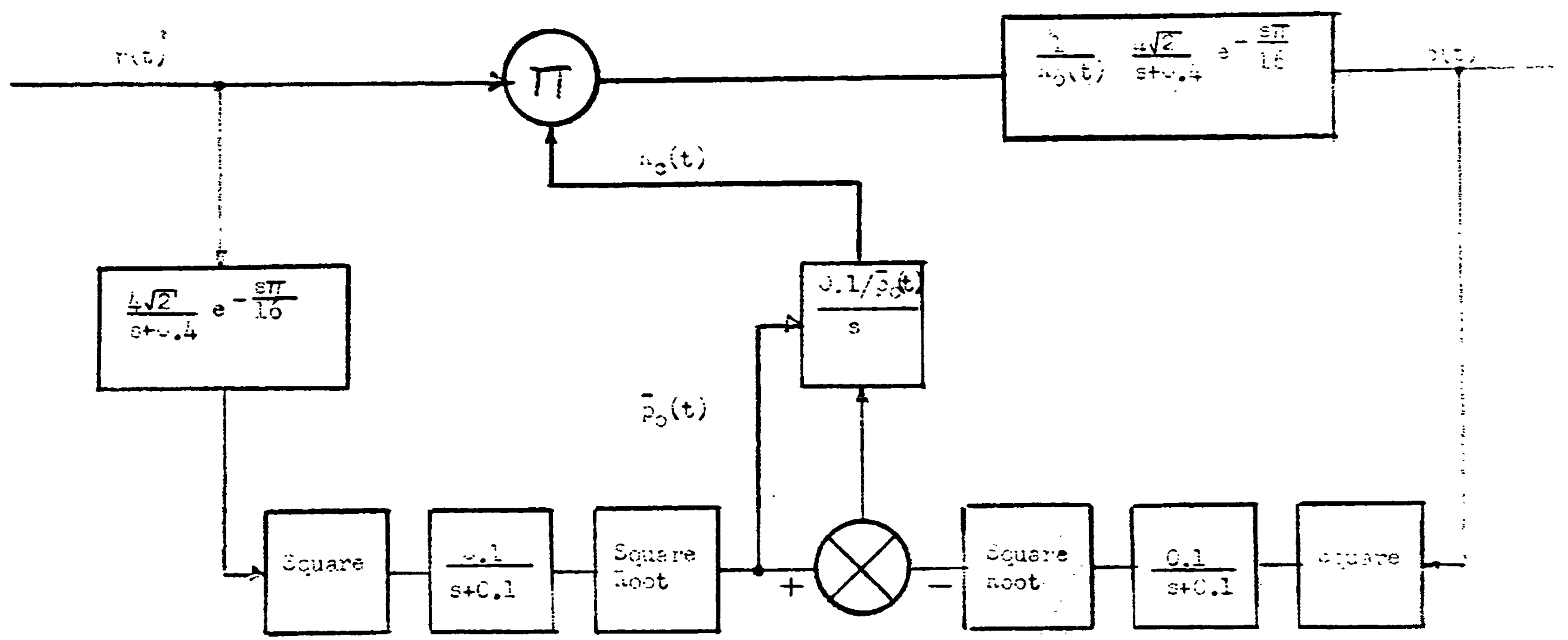

Figure 5 -6 Control System for Impossible Problem control system to the remaining variations. The advantages of each of the two systems is therefore incorporated into the control system.

Chapter 6

Processes With Two Variable Coefficients

The previous chapter deals only with processes that contain one variable coefficient in their characterizing function. These constitute the simplest examples of variable processes in the class being considered. Next in order of complexity are processes that contain two variable coefficients. These represent problems which are of more practical significance. Three processes from this class are investigated. The first is a process with a characterizing function that contains a variable real pole and a variable gain. The second is a process with a characterizing function that contains two variable real poles; the third is a process with a pair of variable complex poles. This last process requires a slight modification in the adaptive circuitry to facilitate analysis.

6.1 A Process With A Variable Pole and A Variable Gain

The characterizing function for a process that contains a variable real pole and a variable gain is

$$G\,(s,t) = \frac{K_m\; G_1(s)}{K_{10}(t)\;[s + a\; K_{20}(t)]} \tag{6-1}$$

The characterizing function for a tandem compensator for this process is

$$G_c(s,t) = \frac{K_n\; K_{1c}(t)\;[s + a\; K_{2c}(t)]}{K_m\;(s + b)} \tag{6-2}$$

This compensator has two variable gain terms and thus it requires two adjusting loops; the selection of the filters for these adjusting loops constitutes the only new design problem.

For the characterizing functions shown "a" is again taken to be the largest value that the pole achieves; $K_{20}(t)$ and $K_{2c}(t)$ are thus both constrained to lie between zero and one. In addition, the constants K_m and K_n are respectively the smallest and desired values of the process gain; hence $K_{10}(t)$ and $K_{1c}(t)$ are also constrained to lie between zero and one.

Now turning to the system block diagram shown in figure 6-1a it is apparent that for frequencies well above $\omega = a$, gain variations in the forward transmission path depend only on $K_{1c}(t)/K_{10}(t)$. As a result, $F_1(s)$ is chosen to be a high pass filter with low frequency cutoff well above $\omega = a$. A model for the operation of this gain adaptive loop is shown in figure 6-1b. This loop thus operates to compensate the process gain constant and it functions in the same manner as the gain correcting loop considered in the previous chapter; this loop is approximately independent of the behavior of the other loop in the system.

Low frequency gain variations in the forward transmission path of the adapted process depend on

$$[K_{1c}(t)/K_{10}(t)]\ [K_{2c}(t)/K_{20}(t)].$$

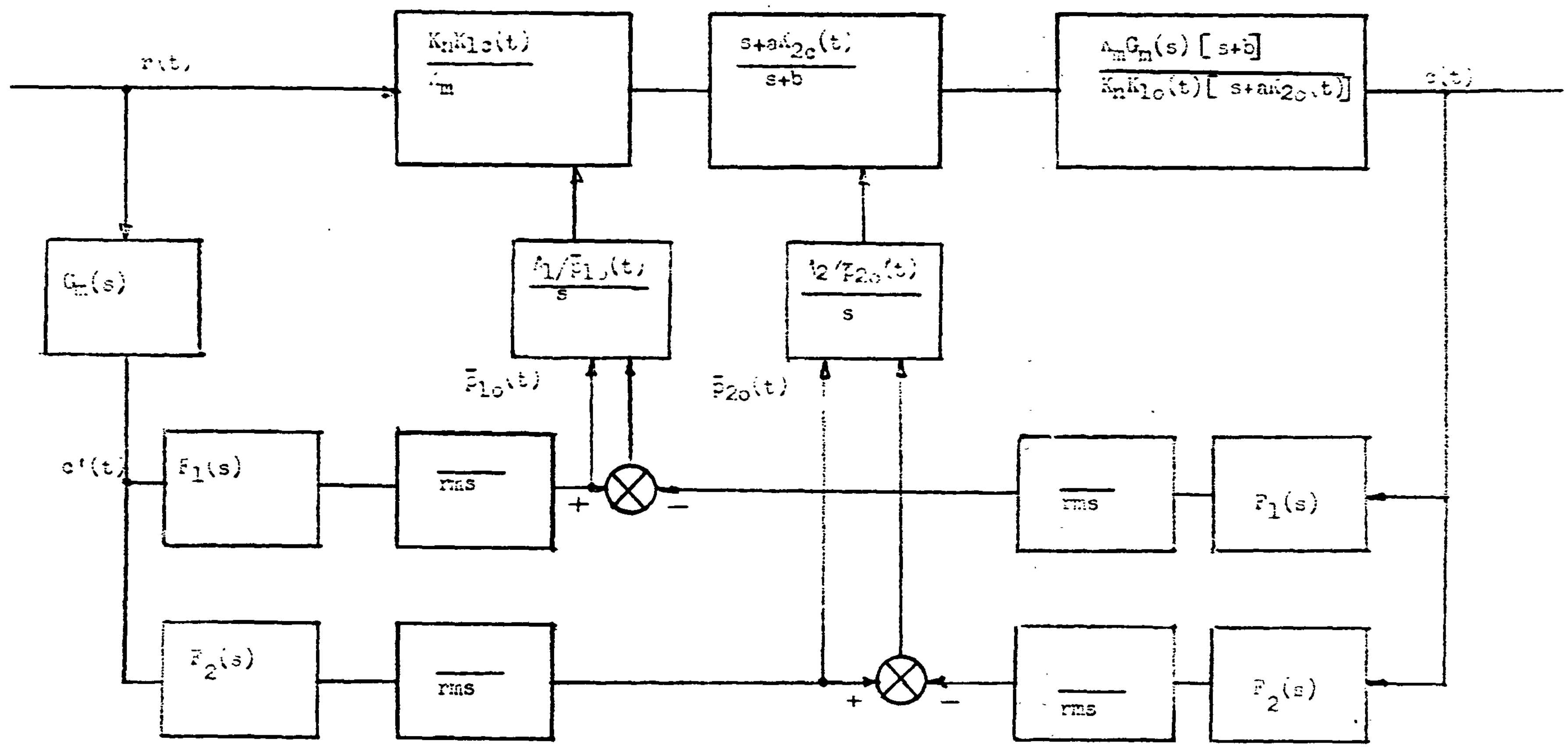

Figure 6-1a Control System for Variable-Gain Variable-Pole Process

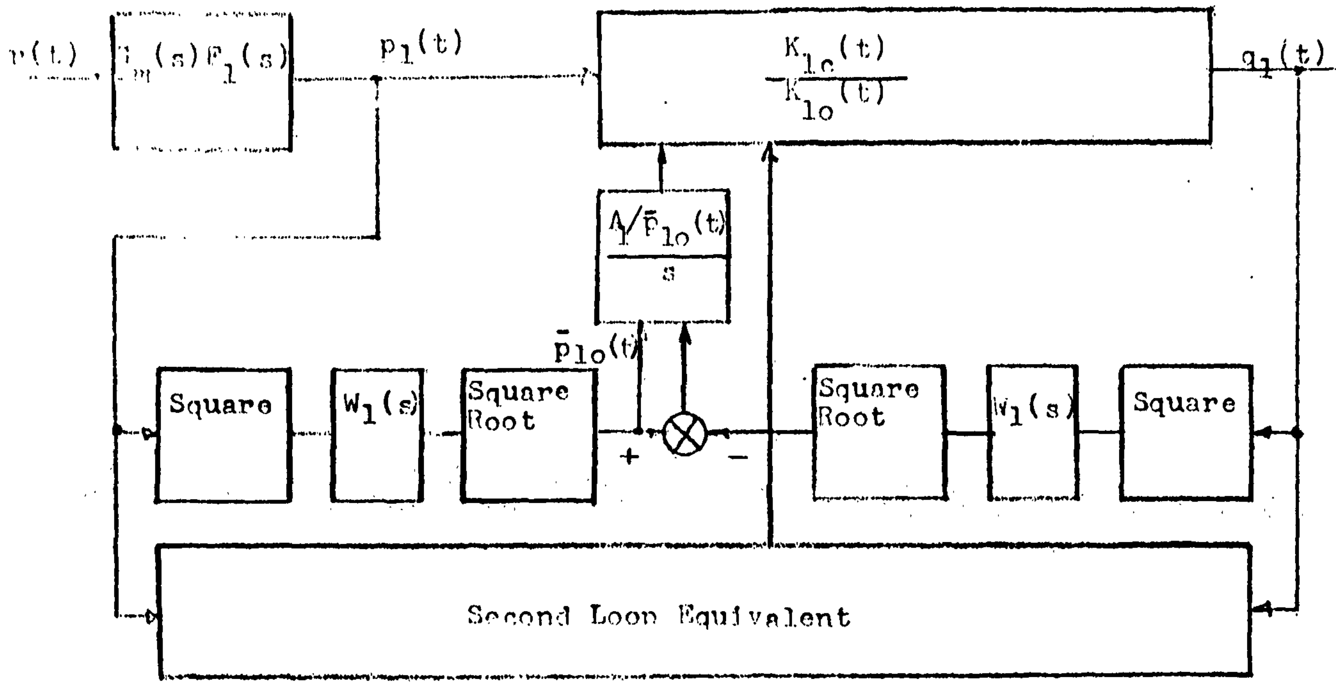

Figure 6-1b Model for Variable Gain Adaptive Loop

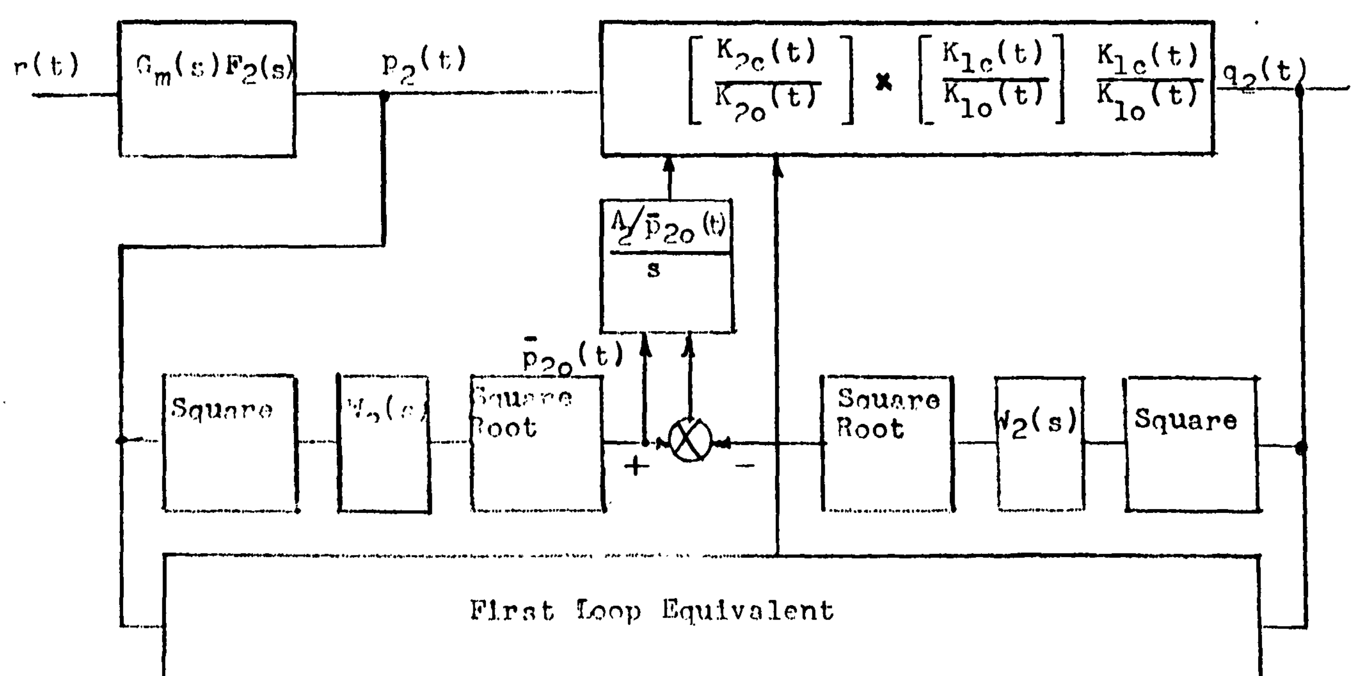

Figure 6-1c Model for Variable Pole Adaptive Loop

$F_2(s)$ is therefore chosen to be a low pass filter with cutoff frequency well below the minimum value of $aK_{20}(t)$. As a result, when the gain constant is approximately compensated, the model for the operation of this second loop is shown in figure 6-1c. The loop thus operates to compensate for pole variations, once the gain compensator has operated, in the same manner as previously considered systems.

Any choice for $F_2(s)$ is acceptable; this is true since the gain at all frequencies is of the proper magnitude to cause pole compensation, once the gain variation is assumed properly compensated. As a result of the gain compensator operating independently of the pole compensator, this assumption is reasonable. Arbitrary choice of $F_2(s)$ however, complicates system analysis. For the system proposed, the behavior of both loops is of the form shown in equation (5-5).

6.2 A Process With Two Variable Real Poles

The characterizing equations for the process and for the compensator, in a problem concerned with two variable real poles, are given by

$$G(s,t) = \frac{G_1(s)}{[s + b\,K_{10}(t)][s + c\,K_{20}(t)]} \tag{6-3}$$

$$G_c(s,t) = \frac{[s + b\,K_{1c}(t)][s + c\,K_{2c}(t)]}{[s + d_1][s + d_2]} \tag{6-4}$$

The constants b and c are the largest values that the respective poles achieve; the terms $K_{10}(t)$, $K_{20}(t)$, $K_{1c}(t)$ and $K_{2c}(t)$ are thus all constrained to lie between zero and one.

A block diagram for the system is shown in figure 6-2a and the forward transmission path of an adaptive loop model is shown in figure 6-2b. Investigation of this transmission path and of the four possible types of Bode Plots (shown in figure 6-3) that correspond to this path, indicate the following choice for $F_1(s)$ and $F_2(s)$: $F_1(s)$ is a low pass filter with cutoff frequency in region #1 of the Bode Plots; $F_2(s)$ is a band pass filler with pass band in region #2 of the Bode Plots. Signals passing thru $F_2(s)$ thus allow for the compensation of the shorter time constant of the process while signals passing thru $F_1(s)$ allow for the compensation of the longer time constant of the process, once the short time constant is compensated.

The selection made implies that the two regions remain disjoint; if this is not true the same approach is still applicable but analysis becomes difficult. For the disjoint case variations in the transmission of signal frequency components in the pass band section of region #2 are proportional to $K_{2c}(t)/K_{20}(t)$. The response of the "short time constant" adaptive loop is, therefore, of the form shown in equation (5-5). Variations in the transmission of signal frequency components in the low pass section of region #1 are proportional to $[K_{1c}(t)/K_{10}(t)]$ $[K_{2c}(t)/K_{20}(t)]$. With the "short

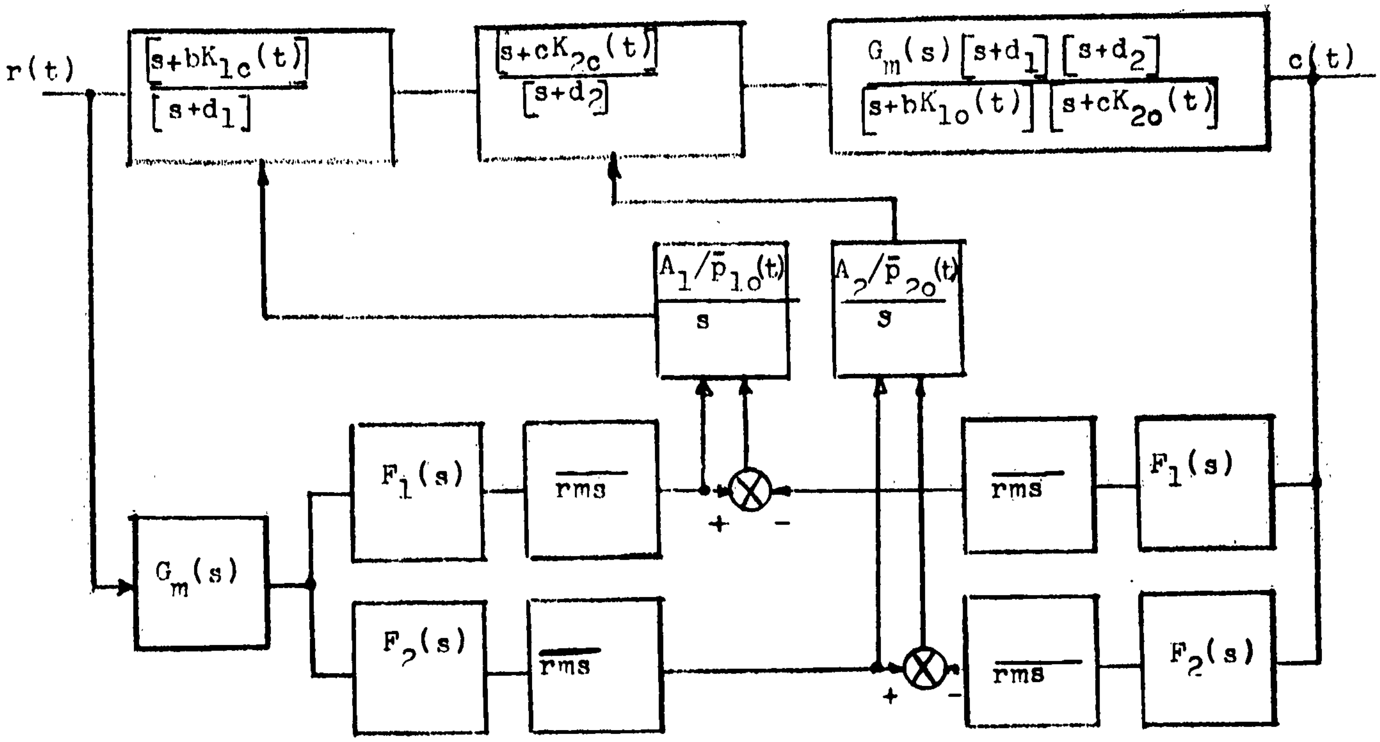

Figure 6-2a Control System for a Two Real Variable-Pole Process

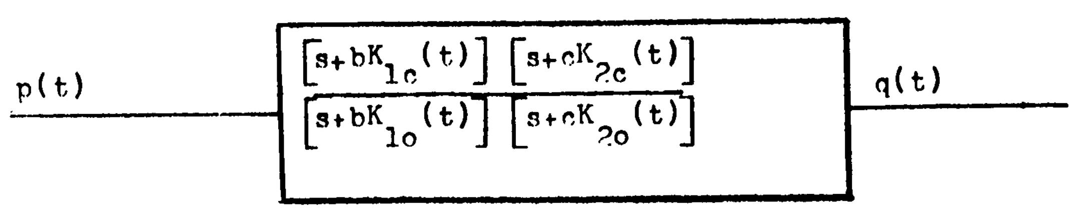

Figure 6-2b Model for the Adaptive Process

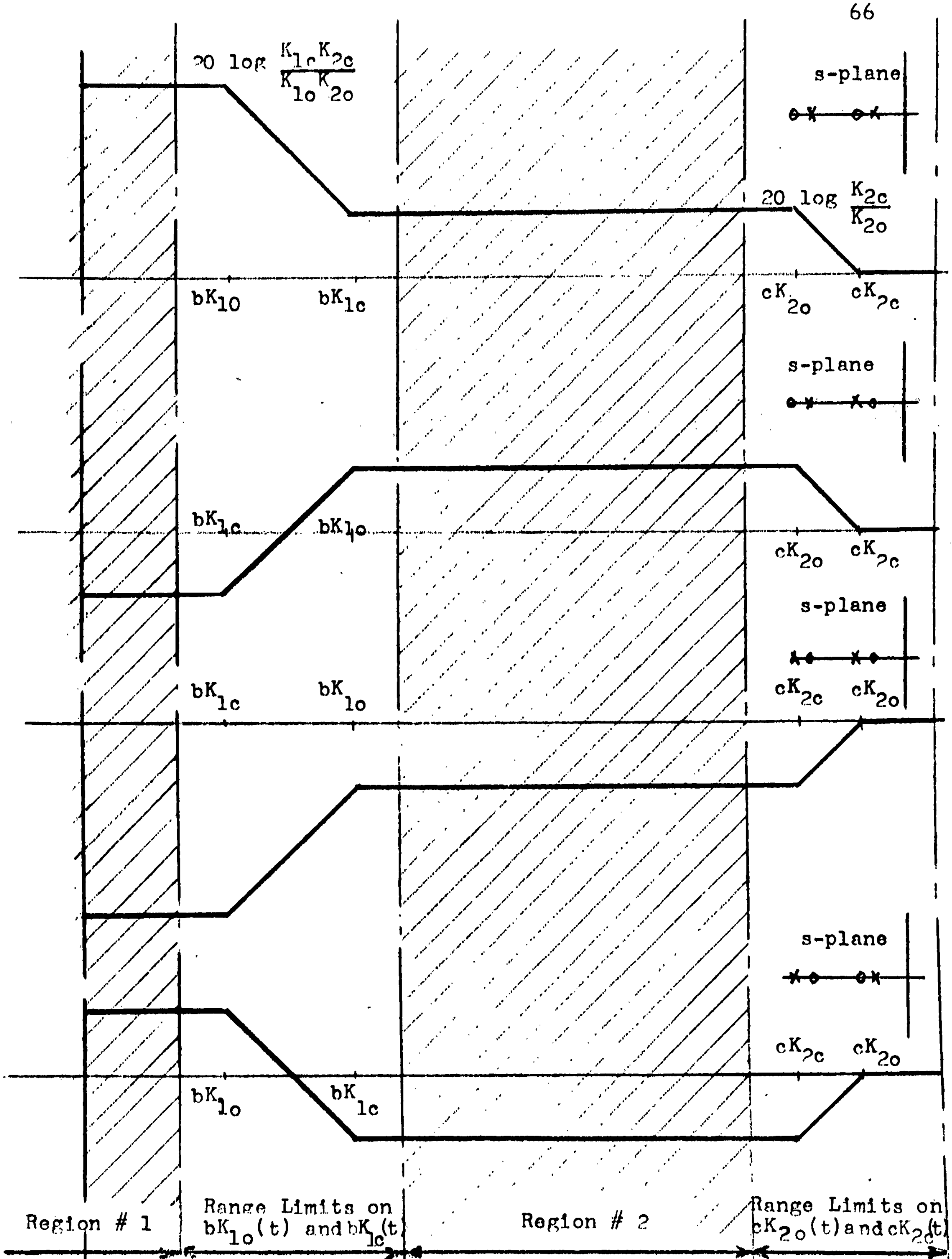

Figure 6-3 Possible Bode Plots

time constant" variation properly compensated, $[K_{2c}(t)/K_{20}(t)]$ is approximately one and thus variation on the low pass region become proportional to $[K_{1c}(t)/K_{10}(t)]$; the response of the "long time constant" adaptive loop is then also of the form shown in equation (5-5). For the non-disjoint case the above statements are approximately true. The choice of filters, however, is restricted by the desire to avoid multi-loop oscillations which may result from inter-loop coupling.

6.3 A Process With A Variable Pair of Complex Poles

One of the more common control system problems is to control a process which contains a variable pair of complex poles in its characterizing function; such a function is given by

$$G(s,t) = \frac{G_1(s)}{s^2 + 2s\,[\xi_o\,K_{10}(t)][\omega_o\,K_{20}(t)] + [\omega_o\,K_{20}(t)]^2} \qquad (6\text{-}5a)$$

The characterizing function of the tandem network used to compensate this process is given by

$$G_c(s,t) = \frac{s^2 + 2s\,[\xi_o\,K_{1c}(t)][\omega_o\,K_{2c}(t)] + [\omega_o\,K_{2c}(t)]^2}{s^2 + 2s\,\xi_n\,\omega_n + \omega_n^2} \qquad (6\text{-}5b)$$

A block diagram of the adaptive process is shown in figure 6-4; a model of this process is shown in figure 6-5a.

For the forward transmission path of the model, the gain for low frequency signal components depends on

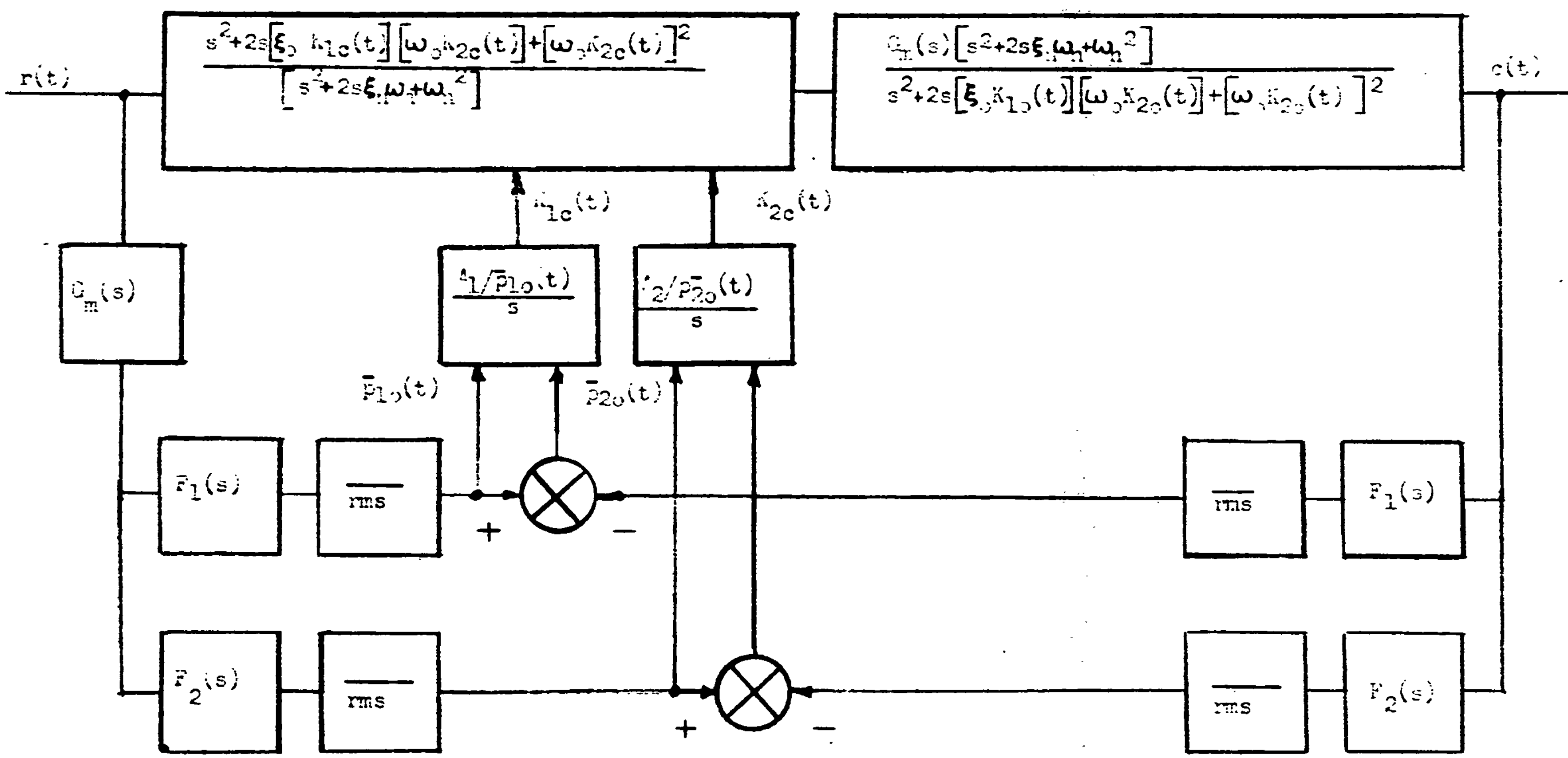

Figure 6-4 Control System for a Variable Complex-Poles Process

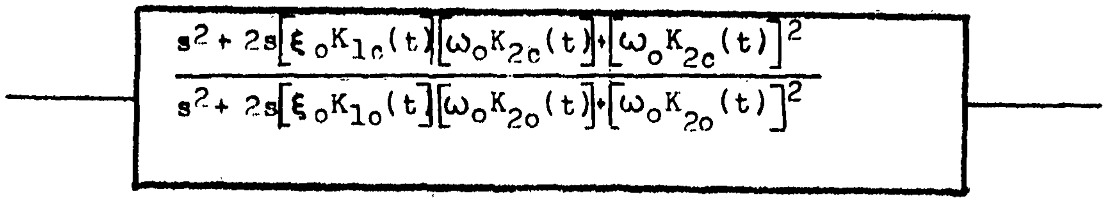

Figure 6-5a Model for the Adaptive Process

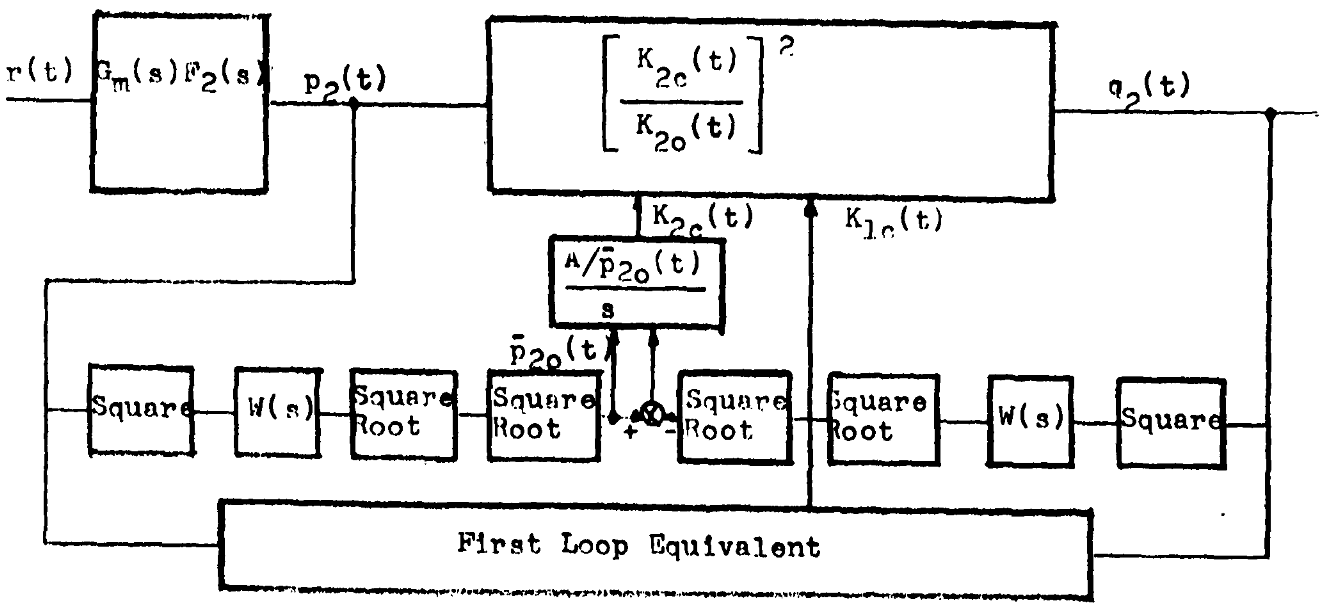

Figure 6-5b Model for the Operation of the Natural Frequency Adaptive Loop

$[K_{2c}(t)/K_{20}(t)]^2$; a low pass filter with cutoff well below the minimum value of $\omega_o \ K_{20}(t)$ is, therefore, chosen for $F_2(s)$. A model for the operation of this "natural frequency" adaptive loop is shown in figure 6-5b. This loop is a modification of the standard loop since a double square root operation is indicated. The double square root operation is used to maintain both an analyzable system and a linear dependence of pole position upon gain value. The operation of this loop is independent of the operation of the "damping" adaptive loop. The response of this "natural frequency" adaptive loop depends on $[K_{2c}(t)/K_{20}(t)]$ and is thus of the general form of equation (5-5).

With the variations in the natural frequency properly compensated, the gain of the forward transmission path of the model, at all frequencies, is of proper magnitude to cause compensation for variations in the damping ratio. As a result, any choice for $F_1(s)$ is acceptable; analysis of the operation of this "damping" adaptive loop is, however, difficult. For the special case where the percentage variation in the natural frequency is small analysis of this loop is possible. $F_1(s)$ is chosen to be a band pass filter with center frequency at the nominal natural frequency. The gain for signal frequency components in this pass band then depends on $[K_{1c}(t)/K_{10}(t)]$ and the standard adaptive loop with the standard response form (i.e., equation 5-5) apply.

The process considered above degenerates to a one variable-coefficient problem when either $K_{10}(t)$ or $K_{20}(t)$ is fixed. For these special cases simplification of the compensator is possible; in addition the adaptive circuitry used in the general problem is simplified by the removal of the loop that is not required, and the problem is solved.

6.4 Conclusion

The three processes discussed in this chapter are typical of the two variable coefficient problems that exist. Only minor modification is required when one or more of the poles considered is replaced with a zero. As a result, the solution of many of the two variable coefficient problems that exist are similar to the solutions shown. The main exception to this statement is the two variable coefficient problem that is a special case of a more complicated variation problem; an example of this is a process with two pairs of complex poles that each have variable natural frequencies. These problems are considered in the next chapter.

Each of the solutions presented in this chapter contains at least one adaptive loop that operates independently of the other loop. The response for each of these independent loops is given by equation (5-5); this equation shows that the loop response to a step displacement of the variable coefficient is the same as the step response of a low pass R-C network. The time constant of this response, however, depends linearly on the final magnitude of the step. The response of any of the other loops is also given by equation (5-5) under the restricted condition that the independent loop has already properly operated.

Chapter 7

Multi-Variation Processes and System Limitations

At this point in the presentation it is appropriate to consider multi-variation processes. Since processes that contain variable complex poles are among the most difficult to handle, and since the technique presented is particularly suited to these problems, processes containing variable complex poles are chosen for consideration. As an example of a three variable coefficient problem, a process with a characterizing function that contains a variable gain and a pair of variable complex poles is considered. For a problem that contains four variable coefficients, a process with a characterizing function that contains two pair of variable complex poles is used. A process with a characterizing function that contains two pair of variable complex poles and a variable gain is the example chosen for use as a five variable coefficient problem; this is the most complex problem considered.

The allowable regions over which the poles and zeros of a process characterizing function vary, directly affects the analysis and the usefulness of a proposed system. The limitations caused by these variations and the limitations caused by noise are investigated in this chapter. In addition, the problem of stability is discussed.

7.1 A Process With Three Variable Coefficients

The first multi-variation process considered has a characterizing function that contains a variable gain and a pair of variable complex poles. This process is compensated thru the use of a tandem network with a characterizing function that contains a variable gain and a pair of variable complex zeros. Three adaptive loops are used to adjust the compensator.

For the process being considered, the natural frequency of the complex poles is constrained to remain between ω_{01} and ω_{02}, while the damping ratio of the poles varies between ξ and 0. A plot of the possible location of these poles in the s-plane is shown in figure 7-1a. Based upon these restrictions, the three adaptive loops are constructed as follows:

The first loop, denoted as the gain loop, is identical to the gain adaptive loop in section 6.1. It contains a high pass filter with low frequency cutoff well above $\omega = \omega_{01}$; thus this loop is only sensitive to variations in the process gain and responds in accordance with equation (5-5). The second loop, denoted as the natural frequency loop, is identical to the natural frequency adaptive loop considered in section 6.3. This loop uses a low pass filter with high frequency cutoff well below $\omega = \omega_{02}$; with the gain variation compensated this loop is sensitive to variations in the natural frequency

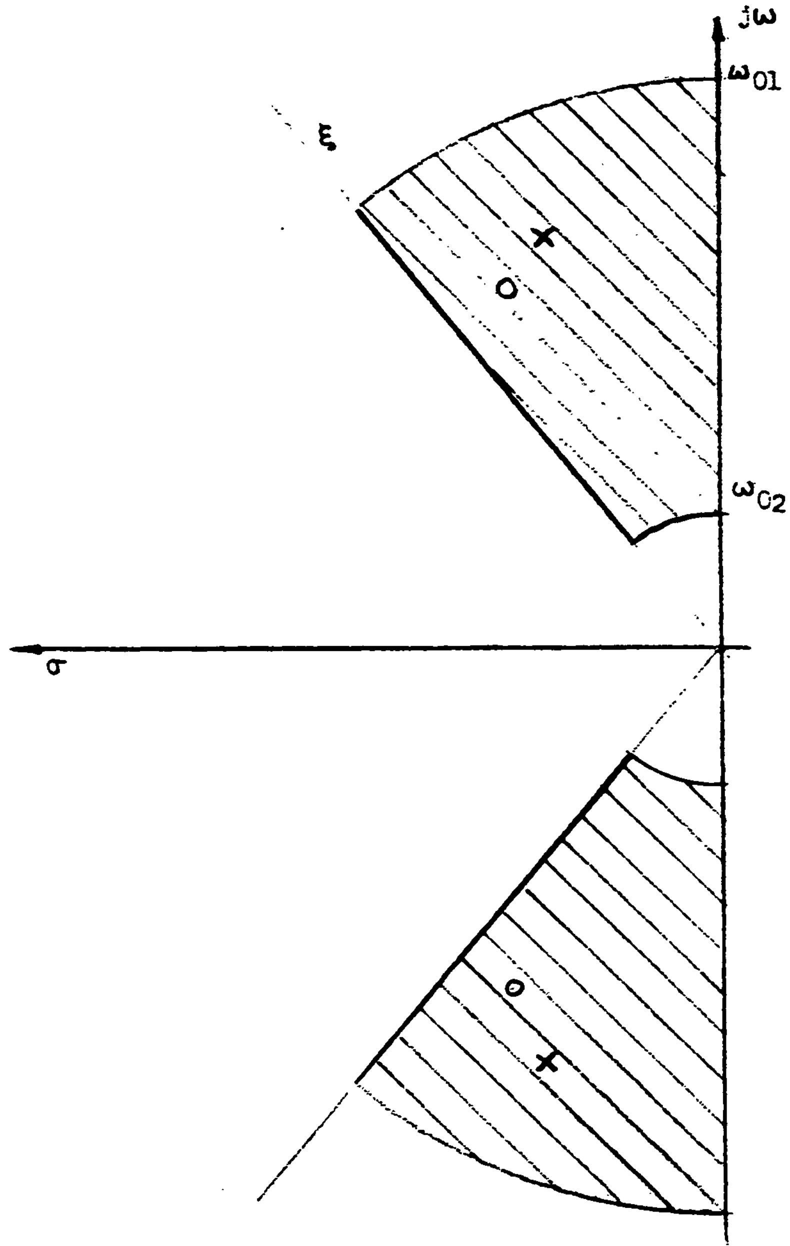

Figure 7-1a Root Locus - One Pair of Poles

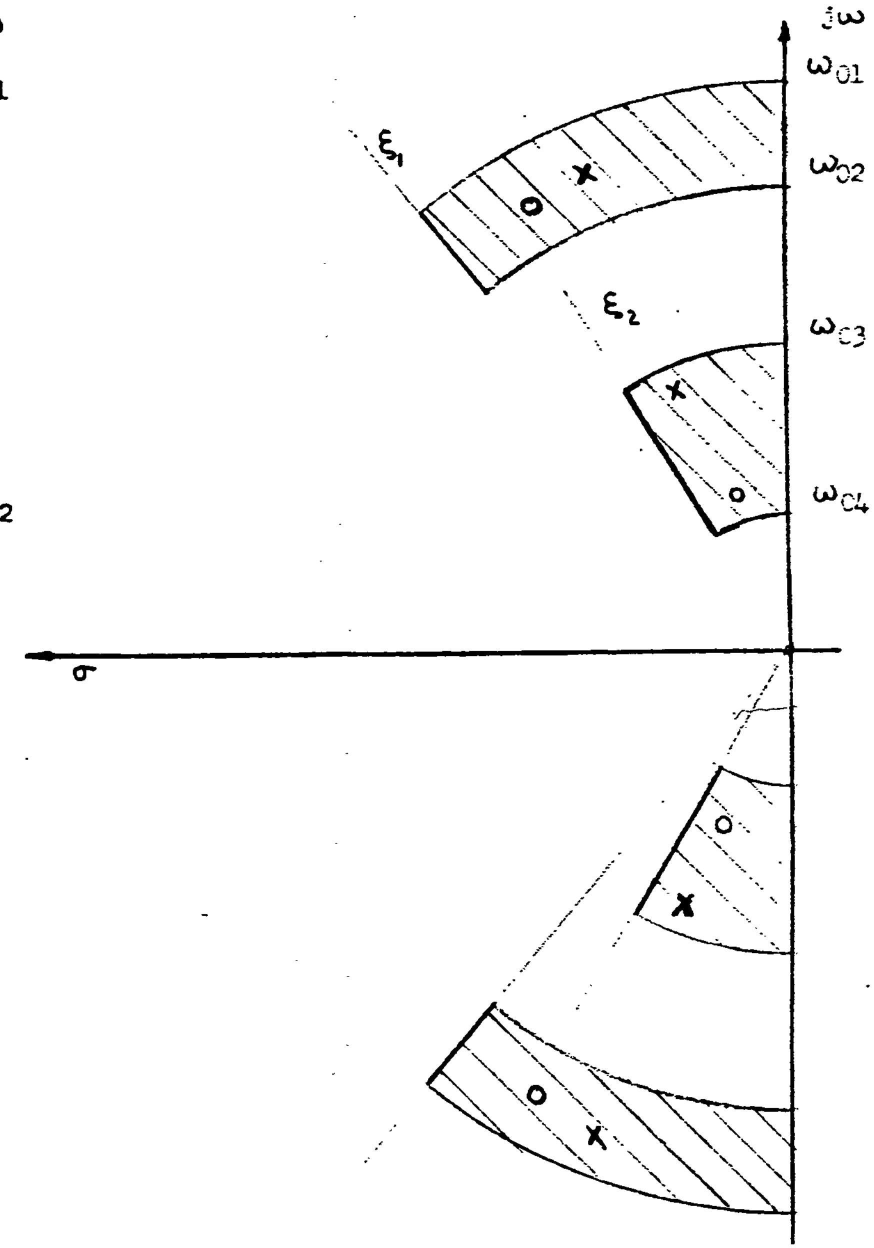

Figure 7-1b Root Locus - Two Pair of Poles

of the variable poles and it too responds in accordance with equation (5-5). The third loop is denoted as the damping loop; it operates to compensate the process for variations in the damping ratio of the variable poles and is identical to the damping ratio adaptive loop considered in section 6.3. For this loop any filter is acceptable. With the gain and natural frequency variations compensated, the adaptive process gain for all signal frequency components is proper to cause compensation by the damping loop. As a result, a band pass filter with pass band between ω_{01} and ω_{02} is selected. This selection insures that no portion of the signal spectrum is used in more than one adaptive loop and thus helps to eliminate interloop coupling.

7.2 A Process With Four Variable Coefficients

The second multi-variation process considered has a characterizing function that contain two pair of variable complex poles. For this process, compensation is accomplished thru the use of a tandem network with a characterizing function that contains two pair of variable complex zeros. These zeros are adjusted on the basis of signals generated in four separate adaptive loops. The design of these loops depends upon the restrictions governing the location of the variable poles in the s-plane.

For a process with variable complex poles located in disjoint regions of the s-plane, as shown in figure 7-1b, two pair of adaptive loops are required. One pair adjusts for variations in natural frequency and the other pair adjusts for variations in the damping ratio. For the poles shown, the natural frequency of the "high frequency pair" remains between ω_{01} and ω_{02} while its damping ratio varies between ζ_1 and 0; the natural frequency of the "low frequency pair" remains between ω_{03} and ω_{04} while its damping ratio varies between ζ_2 and 0. As a result, the filters for the four loop are selected as follows:

The filter in the natural frequency loop for the "high frequency pair" is band pass; its low frequency cutoff is well above ω_{03} while its high frequency cutoff is well below ω_{02}. This loop is thus sensitive to variations of the appropriate natural frequency and responds in accordance with equation (5-5). The filter in the natural frequency loop for the "low frequency pair" is low pass; the cutoff frequency for this filter is well below $\omega = \omega_{04}$. This second loop is then sensitive to variations in both natural frequencies and, when the first loop has compensated for variations in the natural frequency of the "high frequency pair", the second loop then compensates the natural frequency of the "low frequency pair". The response of this loop is then in accordance with equation (5-5).

The remaining two filters are chosen to be band pass; the cutoff frequencies of these filters correspond to the frequency limits on their respective natural frequencies. As a result, when the variations in natural frequency are compensated, the "high frequency pair" damping loop operates to cause compensation.[1] With this completed the last damping loop compensates for any variation in the "low frequency pair" damping ratio. Thus the process is compensated.

7.3 A Process With Five Variable Coefficients

The inclusion of a variable gain term in the process characterizing function above change that four variable coefficient problem into a five variable coefficient problem (e.g. one variable gain and two pair of variable complex poles). The solution to this five variable problem is then simply an extention of the solution above. One additional adaptive loop is added to compensate for the gain term. This loop uses a high pass filter with cutoff frequency well above ω_{01} and is thus only sensitive to gain variations. The response of the loop is given by equation (5-5). Once gain compensation is accomplished the remaining four loops function as described in the previous section.

The characterizing functions for the processes and compensators considered above are shown in table 7-1. In addition, an approximate expression for the error signal in each of the loops is given; these expressions are ordered in accordance with increasing loop dependence.

---

[1]The "low frequency zeros" of the compensator approximately cancel the effect of the "low frequency poles" of the process, for the signal frequency components used by this loop.

	ERROR SIGNALS		
Characterizing Functions	Gain Loop	Natural Frequency Loop	Damping Loop *
$G(s,t)=\dfrac{K_m G_p(s)}{K_{1o}(t)\left[s^2+2s\xi_1\omega_1K_{2o}(t)K_{3o}(t)+\omega_1^2K_{3o}^2(t)\right]}$ $G_c(s,t)=\dfrac{K_N K_{1c}(t)\left[s^2+2s\xi_1\omega_1K_{2c}(t)K_{3c}(t)+\omega_1^2K_{3c}^2(t)\right]}{K_m\left[s^2+2s\xi_n\omega_n+\omega_n^2\right]}$	$\dot{K}_{1c}(t)\doteq A_1\left[1-\dfrac{K_{1c}(t)}{K_{1o}(t)}\right]$	$\dot{K}_{3c}(t)\doteq A_3\left[1-\dfrac{K_{3c}(t)\ K_{1c}(t)}{K_{3o}(t)\ K_{1o}(t)}\right]$	$\dot{K}_{2c}(t)\doteq A_2\left[1-\dfrac{K_{2c}(t)}{K_{2o}(t)}\right]$
$G(s,t)=\dfrac{K_N}{s^2+2s\xi_1\omega_1K_{2o}(t)K_{3o}(t)+\omega_1^2K_{3o}^2(t)}\cdot\dfrac{G_p(s)}{s^2+2s\xi_2\omega_2K_{4o}(t)K_{5o}(t)+\omega_2^2K_{5o}^2(t)}$		$\dot{K}_{3c}(t)\doteq A\left[1-\dfrac{K_{3c}(t)}{K_{3o}(t)}\right]$	$\dot{K}_{2c}(t)\doteq A_2\left[1-\dfrac{K_{2c}(t)}{K_{2o}(t)}\right]$
$G_c(s,t)=\dfrac{s^2+2s\xi_1\omega_1K_{2c}(t)K_{3c}(t)+\omega_1^2K_{3c}^2(t)}{s^2+2s\xi_{n2}\omega_{n2}+\omega_{n2}^2}\cdot\dfrac{s^2+2s\xi_2\omega_2K_{4c}(t)K_{5c}(t)+\omega_2^2K_{5c}^2(t)}{s^2+2s\xi_{n2}\omega_{n2}+\omega_{n2}^2}$		$\dot{K}_{5c}(t)\doteq A_5\left[1-\dfrac{K_{5c}(t)\ K_{3c}(t)}{K_{5o}(t)\ K_{3o}(t)}\right]$	$\dot{K}_{4c}(t)\doteq A_4\left[1-\dfrac{K_{4c}(t)}{K_{4o}(t)}\right]$
$G(s,t)=\dfrac{K_m}{K_{1o}(t)\ \ s^2+2s\xi_1\omega_1K_{2o}(t)K_{3o}(t)+\omega_1^2K_{3o}^2(t)}\cdot\dfrac{G_p(s)}{s^2+2s\xi_2\omega_2K_{4o}(t)K_{5o}(t)+\omega_2^2K_{5o}^2(t)}$	$\dot{K}_{1c}(t)\doteq A_1\left[1-\dfrac{K_{1c}(t)}{K_{1o}(t)}\right]$	$\dot{K}_{3c}(t)\doteq A_3\left[1-\dfrac{K_{3c}(t)\ K_{1c}(t)}{K_{3o}(t)\ K_{1o}(t)}\right]$	$\dot{K}_{2c}(t)\doteq A_2\left[1-\dfrac{K_{2c}(t)}{K_{2o}(t)}\right]$
$G_c(s,t)=\dfrac{K_{1c}(t)\left[s^2+2s\xi_1\omega_1K_{2c}(t)K_{3c}(t)+\omega_1^2K_{3c}^2(t)\right]}{K_m\left[s^2+2s\xi_{n1}\omega_{n1}+\omega_{n1}^2\right]}\cdot\dfrac{K_N\left[s^2+2s\xi_2\omega_2K_{4c}(t)K_{5c}(t)+\omega_2^2K_{5c}^2(t)\right]}{s^2+2s\xi_{n2}\omega_{n2}+\omega_{n2}^2}$		$\dot{K}_{5c}\doteq A_5\left[1-\dfrac{K_{5c}(t)K_{3c}(t)K_{1c}(t)}{K_{5o}(t)K_{3o}(t)K_{1o}(t)}\right]$	$\dot{K}_{4c}(t)\doteq A_4\left[1-\dfrac{K_{4c}(t)}{K_{4o}(t)}\right]$

*Approximation Only Valid Under Restricted Conditions Stated in Text.

TABLE #1

7.4 Stability

With the completion of the discussion of the multivariable coefficient processes considered above, the problem that next requires attention is that of process stability. The questions of adaptive process stability is in general a difficult one to answer; this difficulty arises since the process output depends not only upon the input signal but also upon the process variation involved. As a result, there are two degrees of freedom for the problem and analysis is difficult.

In this dissertation the assumption is made that the allowable variations of the process and the compensator are bounded; this insures that all variable coefficients remain finite. In addition, it is assumed that when the time dependent factors in the characterizing function of the adaptive process cancel [i.e. $T(s) = G(s,t)\ G_c(s,t)$ is not a function of time] the output satisfactorily approximates the input[2]. An adaptive process in the class considered is therefore defined as stable if

$$\lim_{t \to \infty} [G(s,t)\ G_c(s,t)] = G_m(s) \tag{7-1}$$

for any step variation in the coefficients of the process

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[2]This is implied in the choice of a tandem compensator.

characterizing function $G(s,t)$. This definition is identical to the requirement that: given step displacements in the K_{i0} values in accordance with

$$K_{i0}(t) = K_{i1} + \left[K_{i2} - K_{i1}\right] U_{-1}(t) \tag{7-2}$$

then for all i

$$\lim_{t \to \infty} K_{i0}(t) = K_{i2} \tag{7-3}$$

under the restriction that $0 \leq K_{i2} \leq 1$.

Any adaptive loop that behaves according to equation (5-5), satisfies the requirements of equation (7-3); for this loop $K_{ic}(t)$ approaches $K_{i0}(t)$ exponentially. As a result any adaptive process, for which all of the adaptive loops behave according to equation (5-5), is stable. In addition, any adaptive process for which each of the adaptive loops operate to independently track its respective variable coefficient, is stable. Thus an adaptive process for which each adaptive loop is sensitive to only one variable coefficient, and for which any allowable input causes each of the loop error signals to drive toward zero (in response to a step coefficient displacement) is stable. This class of process is defined as "independently" stable.

There are many adaptive processes that are stable for which the operation of the adaptive loops are not independent. From this class a subclass is chosen which is denoted as "sequentially independent." A process that is sequentially independent has the property that a listing of the loops exists for which:

1) The first loop is sensitive to only one coefficient and operates to track this coefficient independently of the other loops.
2) Each loop on the list is sensitive to one and only one variable coefficient that is not sensed by any loop with prior listing; this variable is denoted by the position of the loop on the list.
3) With each of the first M variable coefficients properly compensated, the M+1th loop operates independently to compensate the M+1th variable coefficient.

Thus a sequentially independent process is one that contains a set of loop that will null in sequence. This corresponds to the existance of at least one loop that nulls independently of the other loops, a second loop that nulls independently of all but the first loop, a third loop that nulls independently of all but the first two loops, etc. As a result, every loop of a sequentially independent process will eventually null in response to any step displacement in the variable coefficients of the process characterizing function; a sequentially independent process is therefore stable.

Stability for adaptive process with interdependent adaptive loops is difficult to investigate. For an n variable coefficient process the problem is formulated in terms of two points in an n-dimensional space. Each coordinate axis of this space corresponds to one pair of variable coefficients of the adaptive process [i.e., $K_{1o}(t)$ and $K_{1c}(t)$]. Hence one point in the space corresponds to the state of the process characterizing function and the other corresponds to the state of compensator characterizing function. A stable system is thus one for which the point that corresponds to the compensator approaches the point that corresponds to the process. The behavior of the compensator depends upon the initial states of the process and the compensator, as well as the input signal; hence, the path of the point representing the compensator in n-dimensional space also depends on these quantities.

The question of stability thus reduces to the following question - Given any two points in the n-dimensional space and any input from the allowable class of inputs, does the point corresponding to the compensator approach the point corresponding to the process? Those processes for which the answer is yes, are stable. All other processes are unstable.

One final note concerning unstable processes is appropriate - A process that is unstable does not necessarily function improperly; the problem is that it might function improperly.

7.5 Noise

Any physical process is subject to noise. The problems introduced into the operation of an adaptive process by this noise are therefore investigated. Four noise signal are considered: The first is the noise that accompanies the input signal; this noise is desirable for insuring the operation of the adaptive loops in the absence of actual input signals since the two are indistinguishable. Noise that is internally generated in the process is considered next. Since this noise is reflected to the output of the process without loss of generality, it is considered simultaneously with external noise signals that are applied to the process output (e.g. wind signals on a radar antenna). These are undesirable signals. The last noise signal considered is the one generated in the adaptive circuitry; it too is undesirable.

The effect of the noise that accompanies the input signal is two-fold. First it affects the selection of the desired control system transfer function. This occurs in the design stages of the problem[3]. Next, it functions in the same manner as an input signal and thus enables the adaptive circuitry to function even when the intended input signal is zero. The overall effect of the presence of this signal on the adaptive circuitry is desirable.

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[3]Unless the process is signal adaptive.

For the process being considered, all internally generated noise is reflected to the output. This noise is thus combined with all of the noise signals present at the output that are the result of external sources. The combined signal is then attributed to an equivalent generator and the effect of this generator upon the operation of the adaptive process is apparent; the noise generator effects the output signal adaptive circuitry but does not effect the input signal adaptive circuitry. As a result, the adaptive loop error signals are biased and the compensator nulls incorrectly. A small bias is not a problem, in general.

When the effect of the noise generator is excessive, the operation of the adaptive process is improved using three corrective measures. The first involves the use of feedback around the adaptive process; the effect of the noise generator upon the output and thus upon the operation of the adaptive circuitry is thereby reduced. The second measure involves the use of a fixed bias; this bias is used to offset the average bias that is introduced into the adaptive loop error signals by the noise generator. The last measure involves the use of an interrupter switch. When the input power for an adaptive loop is low enough to allow the output noise generator to cause an appreciable error, operation of that loop is interrupted. One of the last two measures is normally required for all systems where the equivalent noise generator contributes a significant portion of the noise in the output. This inclusion protects against the problem of the compensator readjusting toward an extreme position when the adaptive process input signal is removed.

Since the adaptive loops operate in a balanced mode coherent noise generated in the adaptive circuitry tends to cancel. In addition, since the two circuits are identical, any average value of noise present in the adaptive circuits also tends to cancel. The net effect of noise generated in the adaptive circuitry is therefore minor and is lumped with the noise effects considered above.

7.6 Limitations

The example systems considered in this dissertation are all restricted by the requirement that the variable poles and zeros are confined to disjoint regions in the s-plane. Under this restriction the systems that are designed contain adaptive loops that are either independent or sequentially independent. As a result, the adaptive process involved is stable and its operation is analyzable. For the general system where this disjoint restriction does not apply, a problem in selecting independent or sequentially independent adaptive loops arises. Although such a set of loops may exist for a given process, they are difficult to find: if interdependent loops are used instead, analysis of the operation of the adaptive process is arduous. In addition the possibility of designing an unstable system exists.

The restriction of disjoint allowable regions for the poles and zeros of a process is very reasonable for processes with less than six variable coefficients. For more complex processes, where this requirement is not satisfied, an alternate approach is possible; this approach involves combining all the connected regions into disjoint super-regions and designing loops for the super-regions on the basis of a multiple pole (or zero) at the center of gravity of all the poles and zeros in the region. These center of gravity adaptive loops are then used for average regional compensation or when it is desirable, several single parameter loops are designed to replace a given multiple parameter loop. This is in general a difficult problem.

On several occasions in this presentation the possibility of multi-loop coupling is mentioned. Such coupling occurs if two or more adaptive loops are sensitive to the same variable coefficient or the same signal frequency components. Although the existance of this coupling is not a sufficient condition for instability it does introduce the possibility into the problem.

Systems where the adaptive loops are sequentially independent are stable. In addition, in any system in which the coupling between loops is light, the possibility of instability is remote; this is usually the case in the processes with six or less variable coefficients. As a result, coupling is not a problem for simple systems. In processes containing many variable coefficients, the coupling between adaptive loops is more pronounced; the possibility of instability is therefore greater. The occurrence of instability, however, is not always a problem.

In many cases instability is acceptable. A process with two variable real poles for example, may be adequately compensated by a pair of variable real zeros, even if the zeros do not cancel the poles; proper compensation may be achieved by the system when the center of gravity of the poles and the center of gravity of the zeros concur. This is the case when only the high and low frequency transmissions are of interest. Thus an unstable system performs adequately.

The final limitation discussed, is that imposed by the requirement that the process is linear and slowly time varying. This requirement is necessary for analysis. Non-compliance to these restrictions by a process, however, does not necessarily give a control system that is unsatisfactory. In fact, it is possible that some nonlinear processes, or some processes that are not slowly time varying, perform properly when compensated as described above. Unfortunately analysis under these unrestricted conditions is extremely difficult.

7.7 Conclusion

The multi-variation processes considered in this chapter constitute a fair cross section of the variable process control system problems that are solved using the adaptive process design technique of this dissertation. The five variable coefficient process treated in section 7.3 is as complicated a problem as is normally of practical interest. Since the technique is applicable to more complex systems, the conclusion that the approach is general, is reasonable. In addition, noise and stability constitute two major problems in the design of any control system. Based on the discussion presented in section 7.5 and 7.6, these do not constitute a major drawback. The systems are, in general, stable and not overly sensitive to the problem of noise. These considerations, when added to the reasonable analysis that is required to investigate the operation of the adaptive process, make the technique appear practical. This practicability is borne out by the results of simulations presented in the next chapter. Finally, an investigation of the solutions presented in this chapter testifies to the simplicity of the technique presented.

Chapter 8

Results of Computer Simulations

Solutions are presented in the preceding chapters to many variable process problems. These solutions are all very attractive on a theoretical basis; there remains, however, two additional questions that require answers. These are: How well does the actual performance of a system compare with the theoretically predicted performance, and how does the system perform when the circuitry is slightly modified?

To answer the first question three adaptive processes are investigated. These include a variable gain process, a process with a variable gain and a variable real pole, and a process that contains a variable gain and two pair of variable complex poles. The second question is answered by modifying the circuitry in the solution of the process with a variable gain and a variable real pole, and observing the modifications in the behavior of the compensator.

The operation of each of the adaptive processes listed above was simulated on a digital computer and the results presented are based on the data thus obtained. All input signal are either sinusoids or sums of sinusoids; this simplifies the simulation but is not necessary for proper operation. The AGC amplifiers are omitted since the average weighted input power is approximately constant; this further simplifies the simulation.

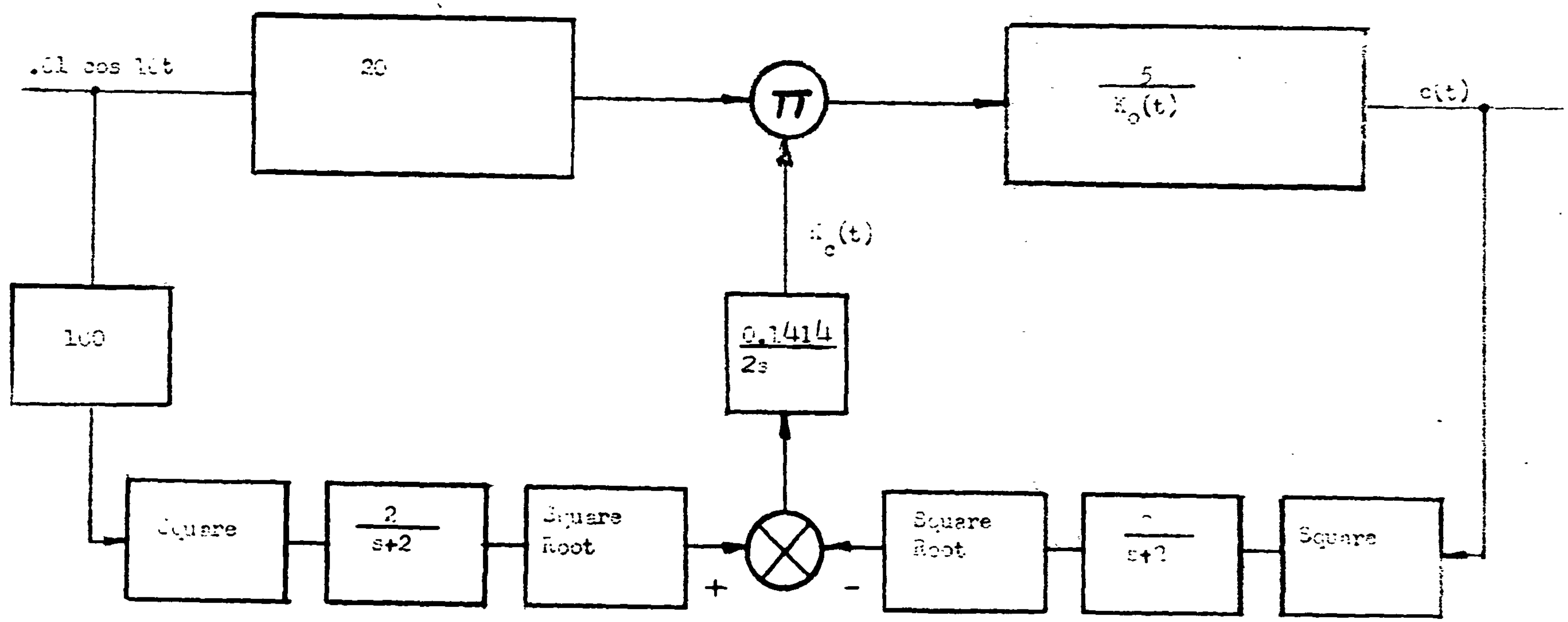
8.1 The Variable Gain Process

The first adaptive process simulated is that shown in figure 8-1; the process is a variable gain amplifier. For this circuit two variations are investigated. The first involves a step displacement in the gain of the amplifier and the second involves a gain that increases with time. The results of section 5.1 are used to calculate a theoretical behavior for the adaptive process.

The results of the simulation are presented in figures 8-2 and 8-3. The curves corresponding to the theoretically predicted response and the perfect response for the compensator, are also presented for comparison. From these curves it is apparent that the actual response and the predicted response differ only slightly; this discrepancy is attributed in part to the omission of the AGC amplifier and in part to the delay in the averaging circuits.

8.2 A Process with Variable Gain and a Variable Real Pole

The second adaptive process simulated is shown in figure 8-4. For this process both the gain term and the time constant suffer a step displacement. The responses of the two variable gains of the compensator to this disturbance are presented in figures 8-5 and 8-6. For comparison the perfect and predicted responses are also presented.



a) $K_o(t) = 1/2\ U_{-1}(t)$
b) $K_o(t) = 1/\ 1+0.1[tU_{-1}(t)]$

Figure 8-1 A Variable Gain System

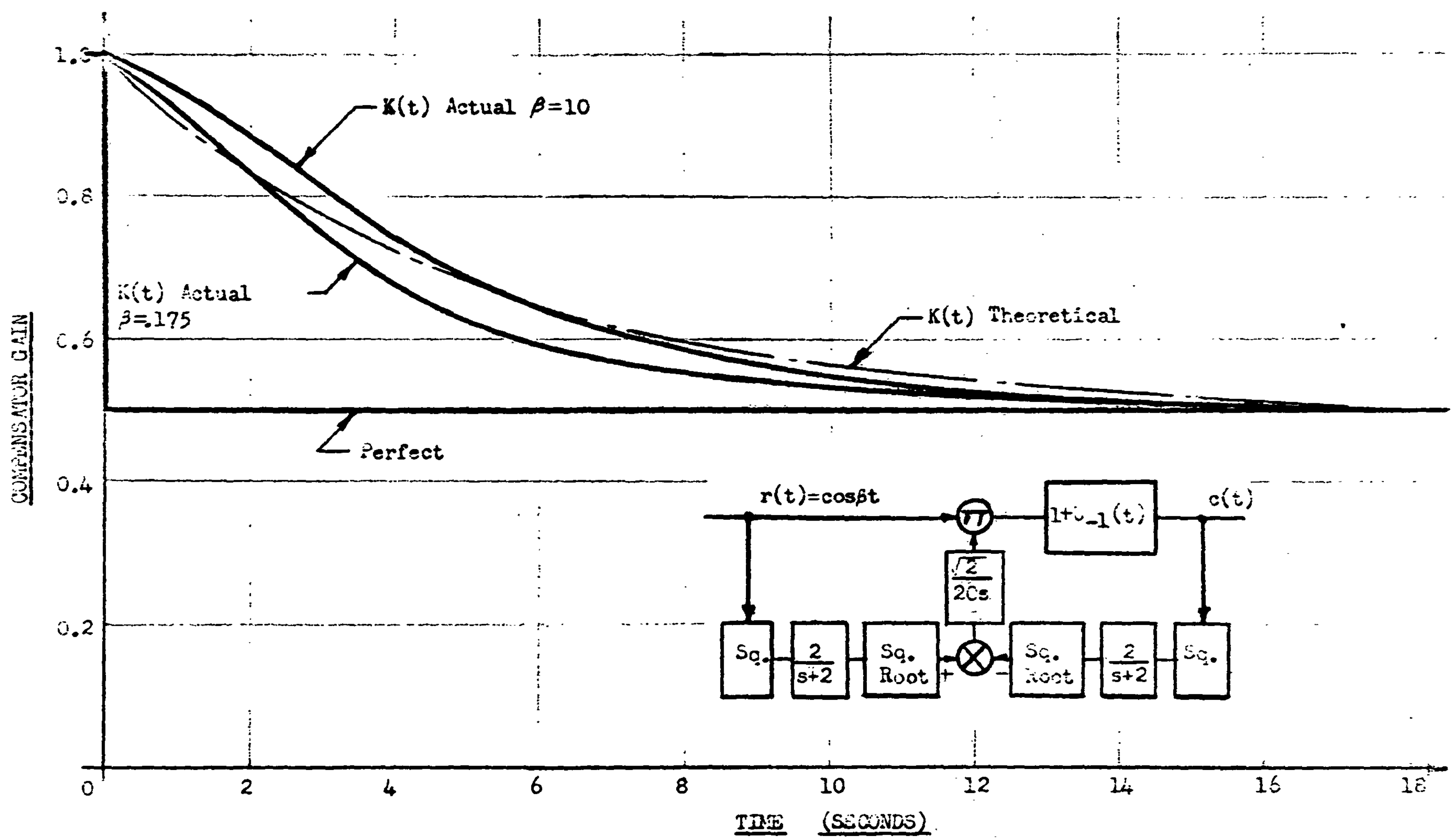


Figure 6.2 System Operation for Step Gain Displacement

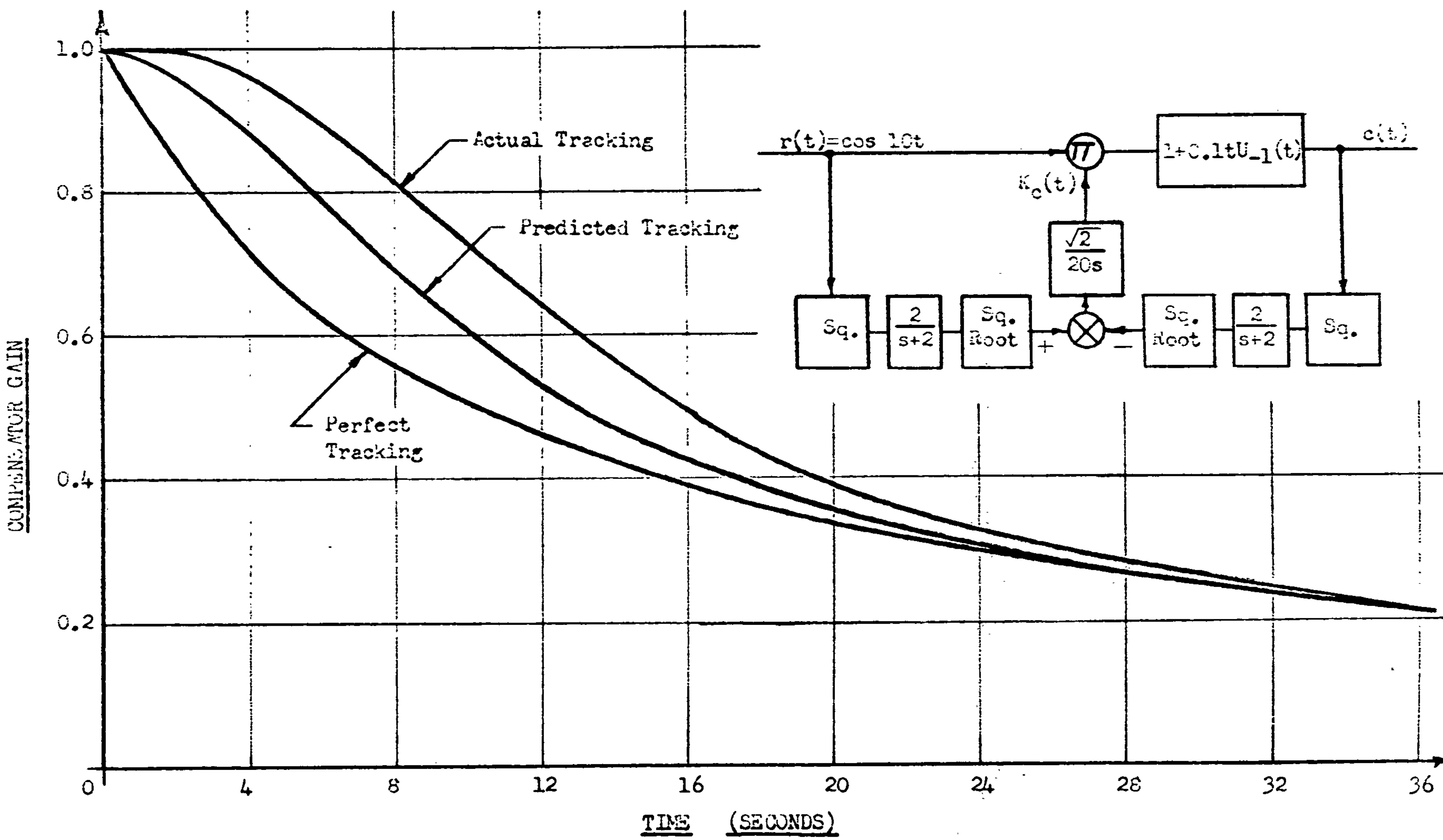
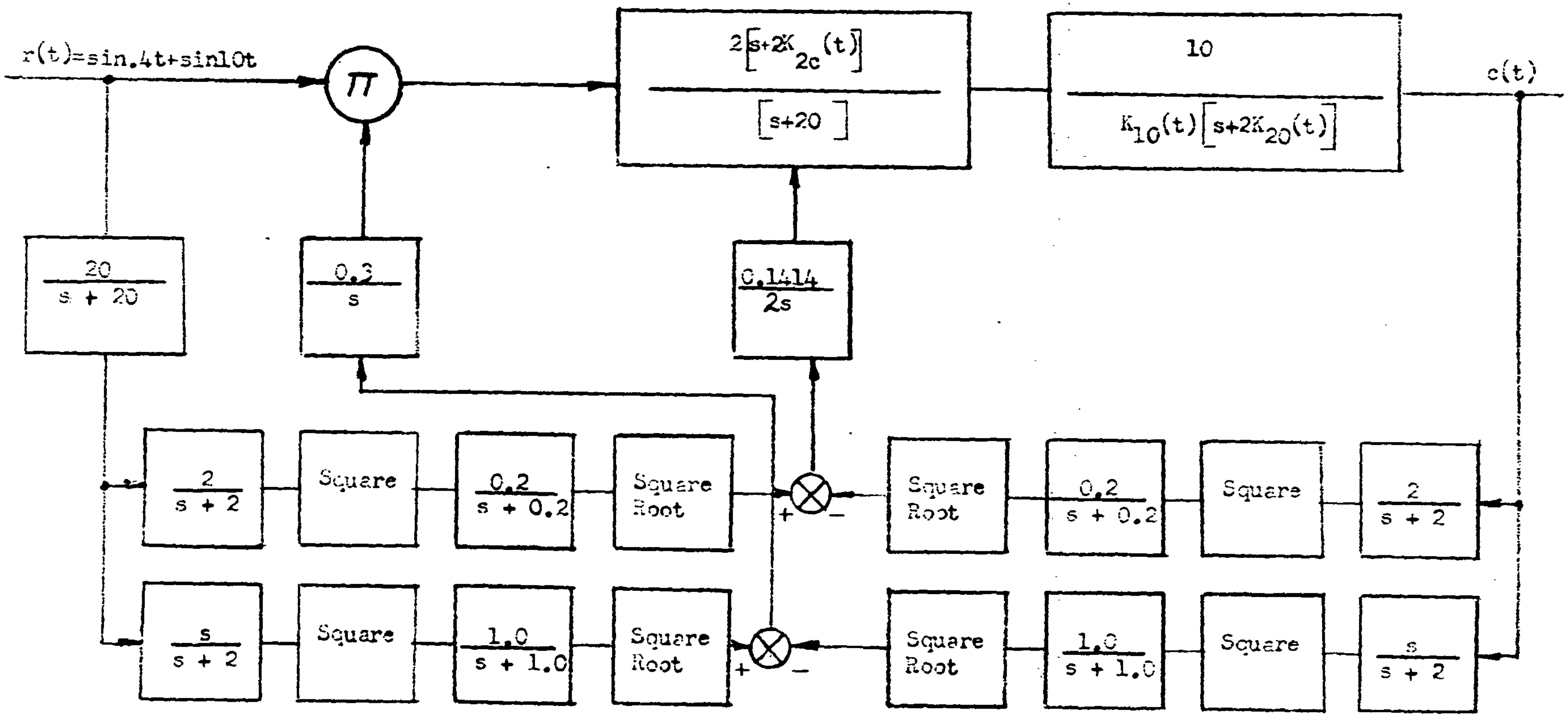


Figure 8.3 System Operation for Ramp Gain Displacement



$K_{1C}(t)=1.00 - 0.25U_{-1}(t)$ $K_{2C}(t)=1.00 - 0.50U_{-1}(t)$

Figure 8.4 A Variable-Gain Variable-Pole System

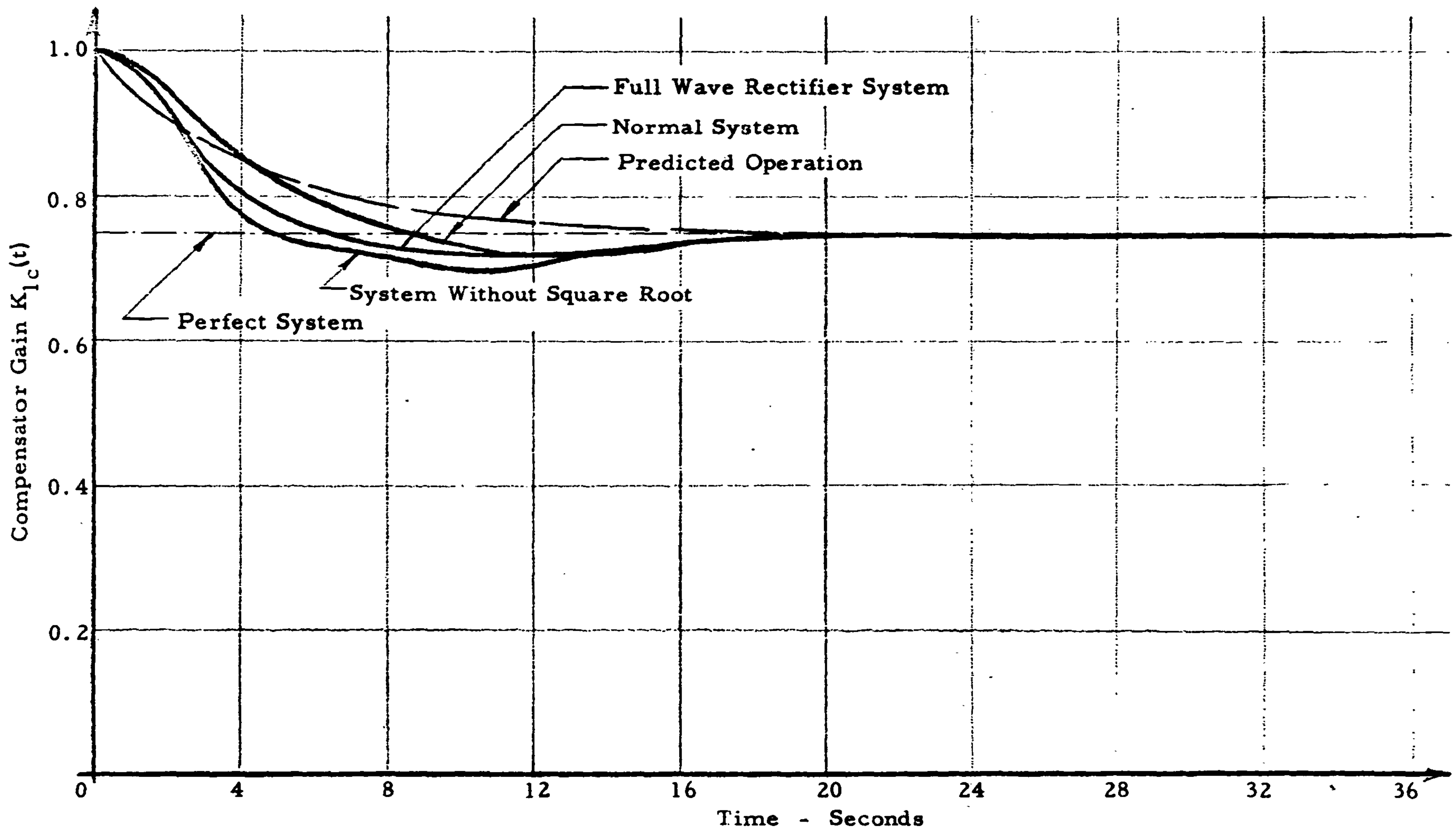


Fig. 8.5 Behavior of compensator gain $K_{1c}(t)$.

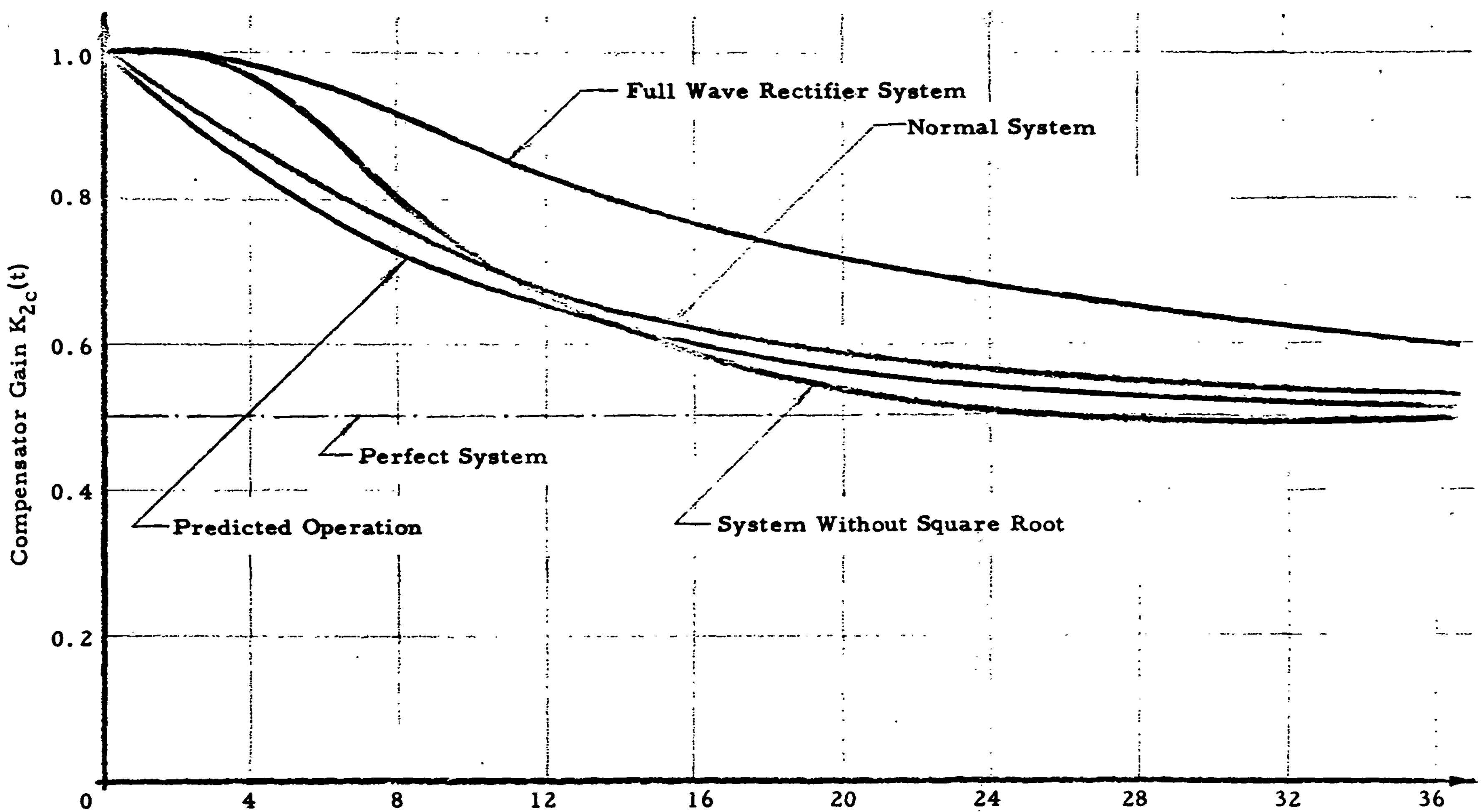


Fig. 8.6 Behavior of Compensator Gain $K_{2c}(t)$.

This adaptive process is then modified. The square root operations are omitted and the compensator response is obtained; this is added to figures 8-5 and 8-6. The square operation is then replaced by a full wave rectifier and the response of the compensator is again obtained; this too is plotted in figures 8-5 and 8-6. The effect of changes on the operation is thus displayed; the asymptotic behavior, however, remains approximately constant.

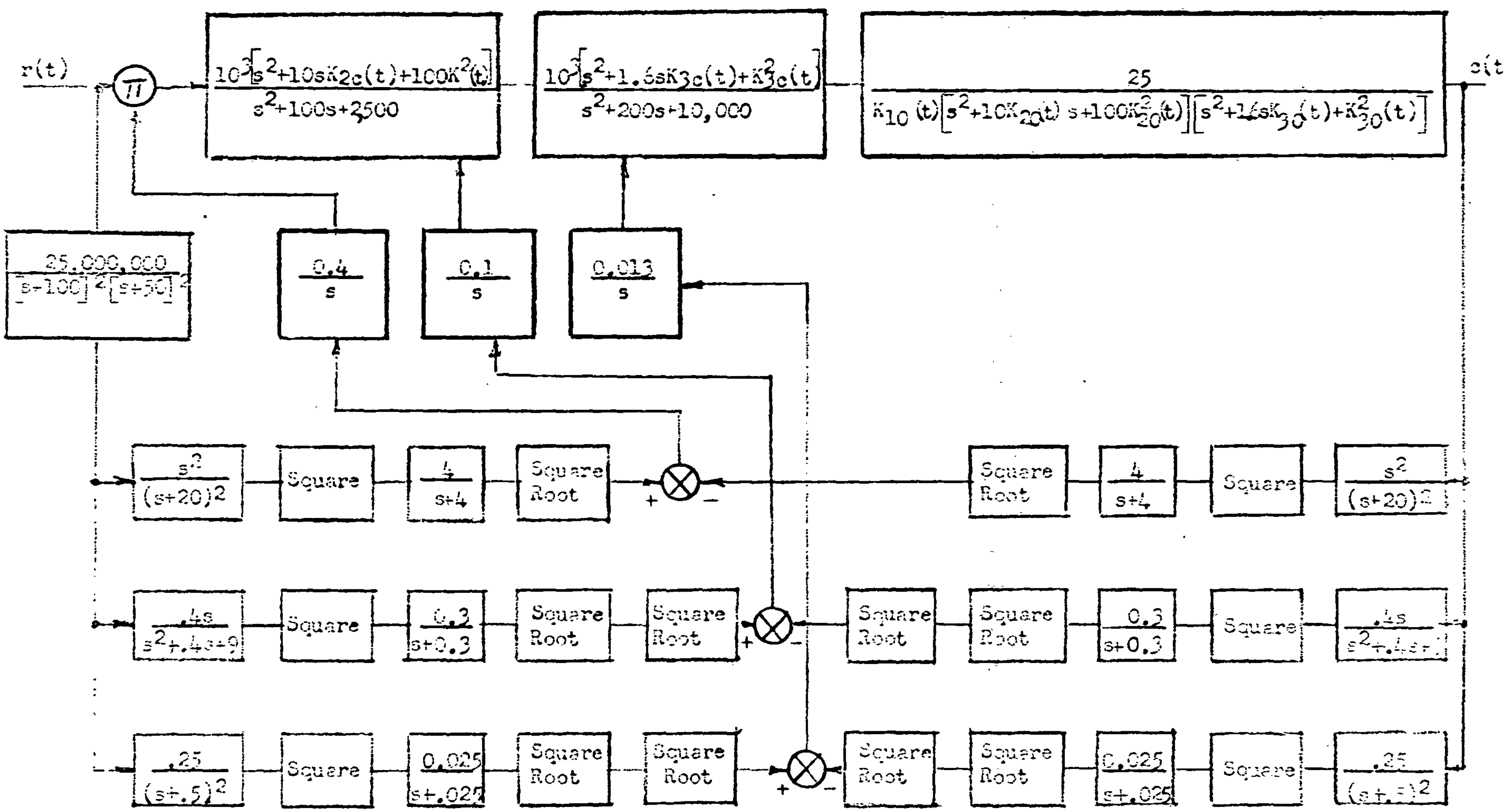
8.3 A Process with a Variable Gain and Two Pair of Variable Comples Poles.

The last adaptive process considered is shown in figure 8-7. The process contains a variable gain and two pair of variable complex poles; these poles have variable natural frequencies. The damping ratio for each pair of poles is fixed.

The three variable coefficients are subjected to step displacements and the responses of the variable gains in the compensator are obtained. These responses are presented in figure 8-8. The theoretically predicted responses for these three gain terms are also presented and the agreement between the two sets of curves is good.

8.4 Conclusion

The results of the computer simulations, for the adaptive processes investigated, indicate that the operation of processes designed in accordance with the technique presented in this dissertation are quite satisfactory. The r(t)=sin 40t + sin 3t + sin 0.25t



$K_{10}(t)=0.25+0.25U_{-1}(t)$ $K_{20}(t)=1.00-0.25U_{-1}(t)$ $K_{30}(t)=1.00-0.25U_{-1}(t)$

Figure 8.7 A Control System for a Process that Contains a Variable Gain and Two Pair of Variable Complex Poles

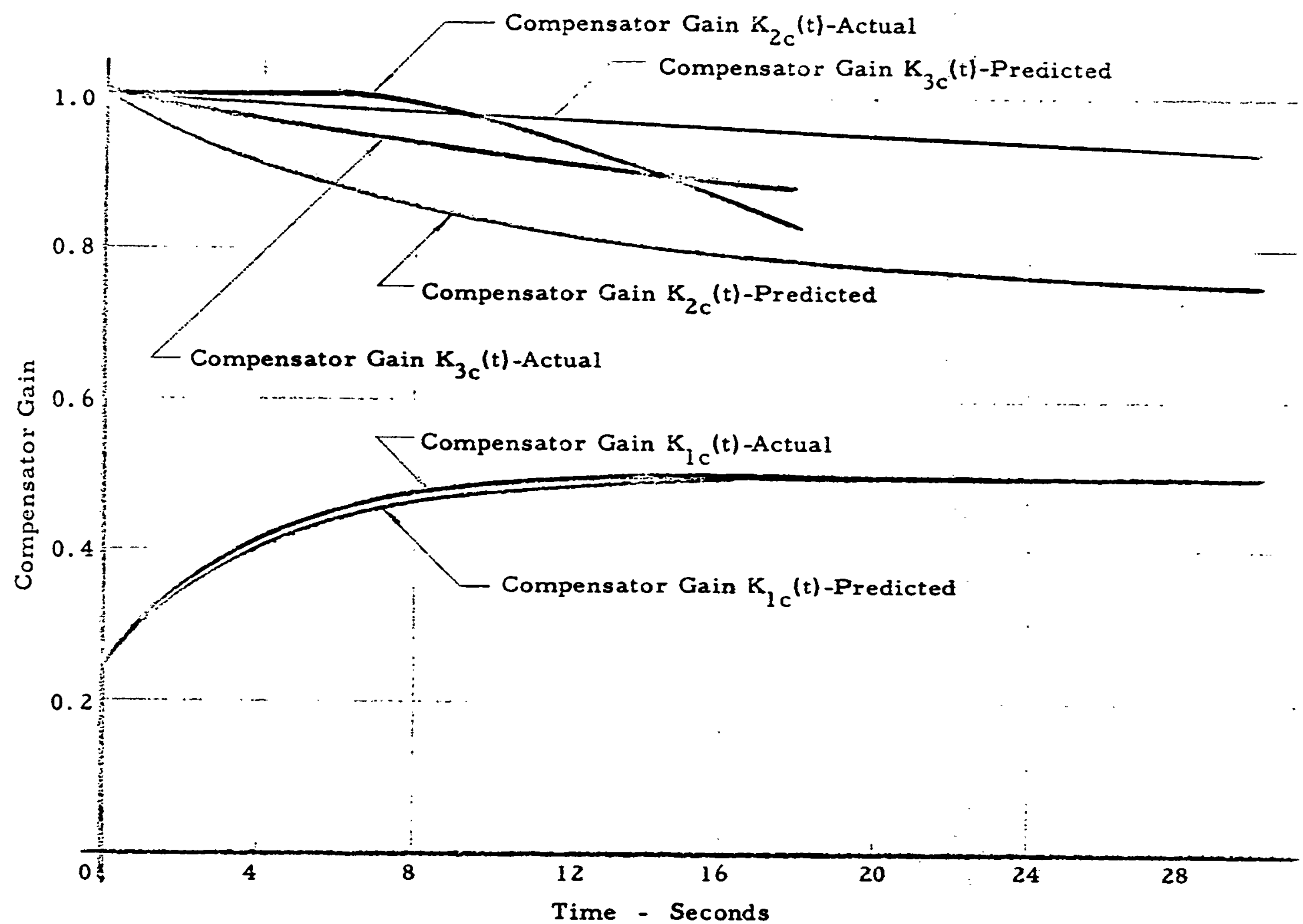


Fig. 8.8 System Operation for a Step Gain Displacement.

agreement between the predicted process response and the actual process response is excellent; comparison of the actual response of the adaptive circuitry with the response required for "perfect" compensation is likewise good. The performance of each of the systems is satisfactory. In additon, the system performance, when subjected to major modifications in the adaptive circuitry, remains acceptable. Removal of the square root operations and replacement of the square operation by a full wave rectifier cause no problems in the operation of the two variable parameter process investigated.

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